MANAGEMENT SCIENCE

COMPLEMENTARY COURSE

IV Semester

BBA

(2011 Admission)

UNIVERSITY OF CALICUT

SCHOOL OF DISTANCE EDUCATION

Calicut University P.O. Malappuram, Kerala, India 673 635

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STUDY MATERIAL

Complementary Course for

BBA

IV Semester

MANAGEMENT SCIENCE

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CHAPTER 1

MANAGEMENT SCIENCE

Management science is a synonymous with operational research. Operation s research provides a quantitative technique or a scientific approach to the executives for making good decisions for operation under control. It provides a scientific approach to problem solving for executive management.

Definitions

Operations research is the application of methods of science to complex problems arising in the direction and management of large systems of man, machine, material and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of the alterative decisions, strategies and control. The purpose is to help management, determine its policy and actions scientifically.

Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control. - P.M. Morse and G.E. Kimball.

This definition suggests that the Operations Research provides scientific methods for an executive to make optimal decisions but does not give any information about various models or methods. But this suggests that executives can use scientific methods for decision-making.

According to Daellenbach and George, “operations research is a systematic application of quantitative techniques and tools which in conjunction with a system approach are applied to solve practical decision problems of an economic or engineering nature.”

According to SL Cook, operations research has been described as a method, an approach, a set of techniques, a team activity, a combination of many disciplines and extension of particular decisions (mathematics, engineering, economics) a new discipline, a vocation, even a religion. It is perhaps some of all these things.”

Operations research is anew discipline. This method utilizes the inter disciplinary team work to solve a complex management problem. An operations research approach seeks to obtain an optimal solution to the problem under analysis. It is a continuous process.

Objectives of operations research

The objective of Operations Research is to provide a scientific basis to the decision maker for solving the problems involving the interaction of various components of an organization by employing a team of scientists from various disciplines, all working together for finding a solution which is in the best interest of the organization as a whole. The best solution thus obtained is known as optimal decision".

Development of operations research

The foundation of Operations Research was during the Second World War. At the time of Second World War, the military management in England invited a team of scientists to study the tactical and strategic problems related to air and land defense of the country. The problem attained significance because the resources available with England were very limited at that time. The objective was to win the war with available scanty resources. The resources such as food, medicines, bullets, labourers etc., were necessary to manage war and for the use of the population of the country. It was essential to decide upon the most successful utilization of the available resources to achieve the objective. It was also essential to utilize the military resources carefully. Hence, the Generals of military, invited a team of experts such as scientists, doctors, mathematicians, business people, professors, engineers etc., and the problem of resource utilization is given to them to discuss and come out with a feasible solution. These specialists had a brain storming session and came out with a method of solving the problem. This method worked out well in solving the war problem. As this method of solving the problem was invented during the war period, the subject is given the name 'operations research'.

After the war scientists who had been active in the military Operations research group made efforts to apply the operations research approach to civilian problems related to business , industry, research development etc. in 1947 the concept of liner programming was developed. Besides linear programming many other techniques of Operations research such as statistical quality control, dynamic programming, queuing theory and inventory theory were developed before the end of 1950. The development of computers helped to apply many Operations research techniques for practical decision analysis.

In 1950's many colleges and universities introduced Operations research in their curricula. Operations research Society of America was founded in 1952. In 1953 Prof.P.C Mahalanobies established Operations research team in the Indian Statistical Institute, Kolkatha for solving problems related to material planning and survey. Operations research society of India was founded in 1957.

Characteristics of Operations research technique

1. Operations research technique is multidisciplinary.
2. It is used to solve complex management problems.

3. It is a continue process.
4. It is a set of mathematical techniques.
5. It is a scientific approach in decision making.
6. It is a team activity.

Scope of Operations Research

The main objective of operations research is to solve complex management problems. It is mainly used in decision problem. A multi disciplinary team used various operations research techniques to solve complex decision problems. The members of the team work together to find a feasible solution which is beneficial for the entire organization. The scope of the operations research involves the following areas.

1. **In defence operations**

   A number of components are involved in military operations. Each component works to achieve maximum gain from its operation. The experts in this field coordinate the entire activities and they utilize their skill to achieve optimum solution.

2. **In industry**

   In an industrial organization there are number of departments. Each department tries to optimize capital investment. HRM department tries to appoint efficient people at minimum cost. There is a conflict between these departments. The application of operations research techniques helps to integrate the activities of various departments to attain the overall objective of the organization. Decision trees, inventory model, linear programming, transportation model, sequencing model, assignment model and replacement model are helpful to the mangers to solve various problems.

3. **In agriculture**

   Operations research techniques are used to select land area for agriculture and the seed of food grains.

4. **In traffic control**

   Queuing theory is used for traffic control.

5. **In hospitals**

   In hospitals we can see lengthy queues. This problem can be solved by the application of operations research techniques.

Phases of operations research

An operations research analyst has to follow certain sequential steps to solve the problems.

1. **Formulation of the problem**

   The Operations Research analyst or team of experts first have to examine the situation and clearly define what exactly happening there and identify the variables and constraints. Similarly identify what is the objective and put them all in the form of statement. The statement must include a) a precise description goals or objectives of the study, b) identification of controllable and uncontrollable variables and c) restrictions of the problem. The team should consult the personals at the spot and collect information, if something is beyond their reach, they have to consult duty engineers available and understand the facts and formulate the problem.

2. Identify the variables and constraints.

   The objective function faces certain constraints. These are to be identified.

3. Establish the relationship between the variables and constraints by constructing the model.

4. Identify the possible alternative solutions.

5. Give an optimal test to basic feasible solution.

6. Select the optimal solution.

7. Install, test and establish the solution model.

8. Establish controls, implement and maintain the solution.

Techniques of Operations Research

Operations Research Models

1. **Iconic Models:** These models are scaled version of the actual object. They explain all the features of the actual object.
2. **Analogue Model:** In this model one set of properties are used to represent another set of properties. Many a time we represent various aspects on graph by different colours or different lines all these are analog models.
3. **Symbolic Models or Mathematical Models:** In these models the variables of a problem is represented by mathematical symbols, letters etc. To show the relationships between variables and constraints we use mathematical symbols.
4. **Descriptive model:** The descriptive model simply explains certain aspects of the problem or situation or a system so that the user can make use for his analysis.

Predictive model: These models basing on the data collected, can predict the approximate results of the situation under question.

5. **Prescriptive models**: Prescriptive models prescribe the courses of action to be taken by the manager to achieve the desired goal.
6. **Deterministic Models:** In this model the operations research analyst assumes complete certainty about the values of the variables and the available resources and expects that they do not change during the planning horizon.
7. **Probabilistic or Stochastic Models:** In these models, the values of variables, the pay offs of a certain course of action cannot be predicted accurately because of element of probability. It takes into consideration element of risk into consideration.

Techniques

1. **Linear Programming:**

   This model is used for resource allocation when the resources are limited and there are number of competing candidates for the use of resources. The model may be used to maximize the returns or minimize the costs.

2. **Sequencing**

   When a manufacturing firm has some job orders, which can be processed on two or three machines and the processing times of each job on each machine is known, then the problem of processing in a sequence to minimize the cost or time is known as Sequencing model.

3. **Waiting Line Model or Queuing Model**

   A model used for solving a problem where certain service facilities have to provide service to its customers, so as to avoid lengthy waiting line or queue, so that customers will get satisfaction from effective service and idle time of service facilities are minimized is waiting line model or queuing model.

4. **Decision theory**

   OR technique of Decision theory is applied in the stage of evaluation of alternatives.

5. **Game theory**

   Game theory helps to determine the best course of action for a firm in view of the expected counter moves from the competitors.

6. **Transportation problem**

   The aim of this technique is find out the minimum transportation cost.

7. The assignment problem

This technique is used to assign jobs to efficient and suitable persons at minimum cost.

8. Net work analysis

Program evaluation and review technique and critical path method are powerful tools for planning and control of complex jobs involving a large number of complex activities.

Operations research and modern management

The objective of operations research is to provide a scientific base to the decision maker for solving the problems involving the interaction of various components of an organization by employing a team of scientists from various disciplines, all working together for finding a solution which is in the best interest of the organization as a whole.

Operations Research provides manager mathematical tools, techniques and various models to analyze the problems in hand and to evaluate the outcomes of various alternatives and make an optimal choice. This helps the manger in making better and quick decisions. A manger, without the knowledge of these techniques has to make decisions by thumb rules or by guess work.

Business and organizations frequently face challenging operational problems whose successful solution requires certain expertise in applied statistics, optimization, stochastic modeling or a combination of these areas. The following are the some of the areas where the operation research techniques are applied. Some of the areas are Finance, budgeting, investment, purchase, production, marketing, personnel, research and development.

CHAPTER 2

LINEAR PROGRAMMING PROBLEM

Linear programming is widely used mathematical modeling technique, which is developed to help decision makers in planning and decision making as far as resource allocation is concerned. It is a technique for choosing the best alternatives from a set of feasible alternatives, in situation in which the objective e function as well as constraints can be expressed as linear mathematical functions. Linear programming involves optimization of certain functions called objective function subject to certain constraints. Linear programming technique may be used for solving broad range of problems arising in business, government, industry, hospitals, libraries, etc.

Any linear programming model (problem) must have the following properties:

(a) The relationship between variables and constraints must be linear.

(b) The model must have an objective function.

(c) The model must have structural constraints.

(d) The model must have non-negativity constraint.

Objectives of Linear programming

Linear programming is a quantitative tool for optimal allocation of limited resources among competing activities. The objective of linear programming is maximization of profit or minimization of cost.

Linear programming problem is based on certain assumptions.

It is assumed that the decision maker here is completely certain (i.e., deterministic conditions) regarding all aspects of the situation, i.e., availability of resources, profit contribution of the products, technology, courses of action and their consequences etc.

2. It is assumed that the relationship between variables in the problem and the resources available

i.e., constraints of the problem exhibit linearity. Here the term linearity implies proportionality and additivity. This assumption is very useful as it simplifies modeling of the problem.

3. We assume here fixed technology. Fixed technology refers to the fact that the production requirements are fixed during the planning period and will not change in the period.

4. It is assumed that the profit contribution of a product remains constant, irrespective of level of production and sales.

5. It is assumed that the decision variables are continuous. It means that the companies manufacture products in fractional units. For example, company manufactures 2.5 vehicles, 3.2 barrels of oil etc. This is referred to as the assumption of divisibility.

6. It is assumed that only one decision is required for the planning period. This condition shows that the linear programming model is a static model, which implies that the linear programming problem is a single stage decision problem.

7. All variables are restricted to non negative values (i.e., their numerical value will be ≥0).

Application of Linear Programming

1. Agriculture application

Linear programming can be applied in agriculture planning. Example; allocation of limited resources such as acreage, labour, water supply, working capital etc. in a way so as to maximize net revenue.

2. Military application

It includes the problem of selecting weapons system against the enemy.

3. Production management :

i. ***Product mix:*** A company can produce different products each of which requires the use of limited production resources. The management wants to determine the quantity of each product to be produced, knowing the managerial contribution and the amount of resources to be used. In this case the objective function may be maximization of the total profit or minimization of loss subject to certain constraints.

ii. ***Production planning:*** This deals with the determination of the minimum cost of production over the planning period.

4. Portfolio selection

This involves the selection of specific investment activity among several activities. The objective function is to find the allocation which maximizes the expected return.

5. Profit planning

It involves the maximization of profit margin from investment in plant facilities and equipment, cash in hand etc.

6. Physical distribution

It determines the most economical and efficient manner of allocating manufacturing plants and distribution centres for physical distribution.

7. Job evaluation

Selection of suitable person for a specified job and evaluation of a job in organization has been done with the help of Linear programming technique.

Formulation of Mathematical Model to Linear Programming Program

Formulation of Linear Programming model involves the following steps.

1. Identification of the problem and setting up of objectives.
2. Establish the interrelationship between the variables of the situation.
3. Identification of alternative variables
4. Specification of constraints.
5. Summarizing the problem in a mathematical form.

Illustration 1

A manufacturing company is engaged in producing three types of product M, N and O. the production department produces each day, components sufficient to make 100 units of M, 50 units of N and 60 units of O. the management is confronted with the problem of optimizing the daily production of products in the assembly department, where any 200 man hours are available daily for assembling the products. The following additional; information is available.

Type of product	Profit contribution Per unit of product (Rs)	Assembly time per product (hrs)
M	24	1.6
N	40	3.4
O	90	5

The company has a daily order for 40 units of product M and total of 30 units of product N and O. Formulate this problem as linear programming problem so as to maximize total profit.

Solution

Let x_1 = number of units of product M

x_2 = number of units of product N

x_3 = number of units of product O

Profit contribution per unit of products M, N and O are 24, 40 and 90 respectively.

So the objective function is

$24x_1 + 40\ x_2 + 90x_3$

The objective function is based on certain constraints. They are

Assembly time per product in hours = 1.6. 3.4 and 5 for product M, N and O respectively,.

The constraint can be written as

$1.6x_1 + 3.4x_2 + 5x_3 \leq 200$

The second constraint is the maximum units of production.

M = 200 units, N = 50 units. O = 60 units

It can be written as

$x_1 \leq 100,\ x_2 \leq 50,\ x_3 \leq 60$

The next constraint is order commitment.

40 units of product M, 30 units of product N and O

That is $x_1 \geq 40,\ x_2 + x_3 \geq 30$

The problem can written as

Maximize $Z = 24x_1 + 40\ x_2 + 90x_3$

Subject to

$1.6x_1 + 3.4x_2 + 5x_3 \leq 200$

$x_1 \leq 100,$

$x_2 \leq 50,$

$x_3 \leq 60$

is $x_1 \geq 40,$

$x_2 + x_3 \geq 30$

Non negative restrictions are

$x_1 \geq 0.\ x_2 \geq 0,\ x_3 \geq 0$

Graphical method

Graphical method is used to solve linear programming problem. It involves two variables. Each line is represented by each constraint.

Steps

1. Formulate the problem.
2. All constraints may be written as equality.
3. Draw the curve.
4. Find out the feasible region.

Illustration 2

Solve the following problem graphically.

Maximize $Z = 60x_1 + 40x_2$

Subject to:

$$2x_1 + x_2 \leq 60$$
$$x_1 \leq 25$$
$$x_2 \leq 35$$
$$x_1 \geq 0, x_2 \geq 0$$

Solution

$$2x_1 + x_2 = 60$$
$$x_1 = 25$$
$$x_2 = 35$$
$$x_1 = 0, x_2 = 0$$

1. $2x_1 + x_2 = 60$

Let $x_1 = 0$ then

$2 \times 0 + x_2 = 60$

$x_2 = 60$ (0, 60)

Let $x_2 = 0$ then

$2x_1 + 0 = 60$

$2x_1 = 60$

$x_1 = . 60/2 = 30$ (30, 0)

P = (0, 35); S = (25, 0)

Points	x_1	x_2	$Z = 60x_1 + 40x_2$		
0	0	0	0		
P	O	35	60 × 0 + 40 × 35	=	1400
Q	12.5	35	60 × 12.5 + 40 × 35	=	2150
R	25	10	60 × 25 + 40 × 10	=	1900
S	25	0	60 ×25 + 40 × 10	=	1500

Solution is Q = (12.5, 35) and Z = 2150

Point Q passes through two straight lines.

$$x_2 = 35$$

$$2x_1 + x_2 = 60$$

$$2x_1 + 35 = 60$$

$$2x_1 = 60 - 35 = 25$$

$$x_1 = 25/2 = 12.5$$

Point R passes through two straight lines.

$$x_1 = 25$$

$$2x_1 + x_2 = 60$$

$$2 \times 25 + x_2 = 60$$

$$x_2 = 60 - 50 = 10$$

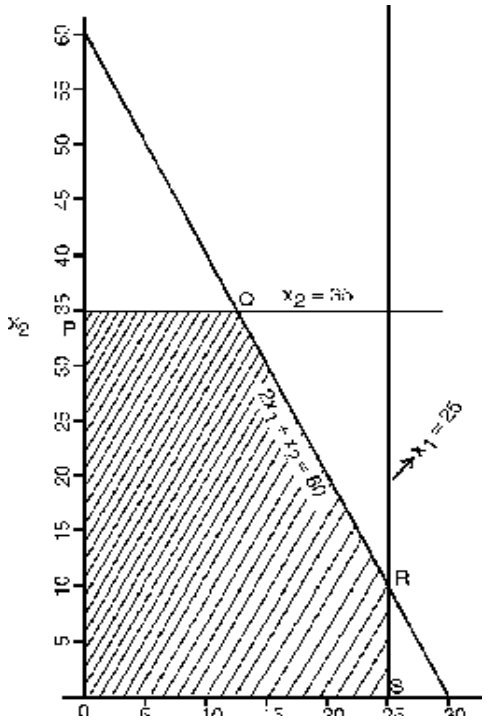


Illustration 3

Minimize $Z = 20x_1 + 40x_2$

Subject to:

$$36x_1 + 6x_2 \geq 108$$

$$3x_1 + 12x_2 \geq 36$$

$$20x_1 + 10x_2 \geq 100$$

$$x_1, x_2 \geq 0$$

Solution

$$36x_1 + 6x_2 = 108$$

$$3x_1 + 12x_2 = 36$$

$$20x_1 + 10x_2 = 100$$

$$x_1 = 0, x_2 = 0$$

1. $36x_1 + 6x_2 = 108$

   Let $x_1 = 0$

$x_2 = 108/6$ (0, 18)

Let $x_2 = 0$

$x_1 = 108/36$ (3, 0)

2. $3x_1 + 12x_2 = 36$

Let $x_1 = 0$

$x_2 = 36/12$ (0, 3)

Let $x_2 = 0$

$x_1 = 36/3 = 12$ (12, 0)

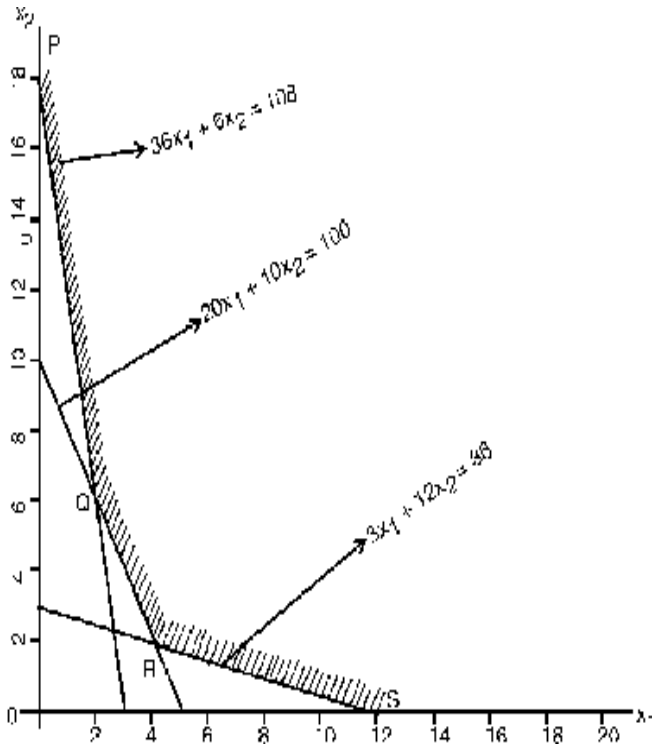
3. $20x_1 + 10x_2 = 100$

Let $x_1 = 0$,

$x_2 = 100/10 = 10$ (0, 10)

$x_2 = 0$, $20x_1 = 100$, $x_1 = 100/20 = 5$

(5, 0)



Points	x_1	x_2	$Z = 20x_1 + 40x_2$	
P	0	18	$0 + 40 \times 18$	= 720
Q	2	6	$20 \times 2 + 40 \times 6$	= 280
R	4	2	$20 \times 4 + 40 \times 2$	= 160
S	12	0	$20 \times 12 + 40 \times 0$	= 240

Thus the optional solution $x_1 = 4$, $x_2 = 2$ and

Z = 160

CHAPTER 3

NETWORK ANALYSIS

Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects. Network analysis is a vital technique in project management. It enables us to take a systematic quantitative structured approach to the problem of managing a project through to successful completion. Moreover, as will become clear below, it has a graphical representation which means it can be understood and used by those with a less technical background. A complex project's data is broken down into its component parts (activities, events, durations, etc.) and plotting them to show their interdependencies and interrelationships.

A network analysis is a generic term for a family of related techniques developed to aid management in the planning and control of projects. These techniques show the inter-relationship of the various jobs or tasks which make up the overall project and clearly identify the critical parts of the project. They can provide planning and control information on the time, cost and resource aspects of a project.

The main objective of network analysis is to establish the overall completion time of projects by calculating what is known as the ***Critical Path.***

Classification of activities:

Activity

This is the task or job of work which takes time and resources. An activity is represented in a network by an arrow, the head indicating where the task ends and the tail where it begins. It normally points left-to-right and is seldom to scale.

Predecessor activity: Activities that must be completed immediately prior to the start of another activity are called predecessor activities.

Successor activity: The activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.

Concurrent activities: The activities that can be accomplished together are known as concurrent activities.

Dummy activity: An activity which does not consume any resource but merely depicts the dependence of one activity on other is called dummy activity. This is an activity which does not consume time or resources, but is merely used to show clear logical dependencies between activities so as not to violate the rules for drawing networks; it is shown by a dotted arrow.

Event:

The beginning and end of activities are called as events. Events are represented by numbered circles called nodes. i j Event start, Event finish

Path & Network:

An unbroken chain of activity arrows connecting the initial event to some other event is called a path. A network is the graphical representation of logically and sequentially connected arrows and nodes representing activities and events of a project. It is a diagram depicting precedence relationships between different activities.

Application of network analysis:

It can be applied in Construction industry, Manufacturing, Research development, administration, Marketing, planning, Inventory planning

Advantages:

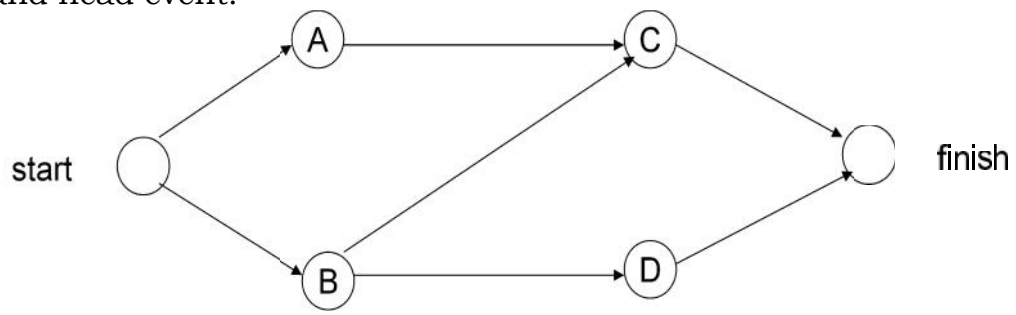
1. Planning & controlling projects
2. Flexibility
3. Designation of responsibilities
4. Achievement of objective with least cost
5. Better managerial control

Guidelines for Network Construction:

- A complete network should have only one point of entry - a START event and only one point of exit - a FINISH event.
- Every activity must have one preceding or 'tail' event and one succeeding or 'head' event (an activity must not share the same tail event and the same head event with any other activities.)
- No activity can start until its tail event is reached.
- An event is not complete until all activities leading in to it are complete.
- 'Loops' i.e. a series of activities which lead back to the same event are not allowed
- All activities must contribute to the network's progression or be discarded as irrelevant (those which do not are termed 'danglers'.)
- Networks proceed from left to right.

- Networks are not drawn to scale i.e. the length of the arrow does not represent time elapsed.
- Arrows need not be drawn in the horizontal plane but unless it is totally unavoidable they should proceed from left to right.
- If they are not already numbered, events or nodes should be progressively numbered from left to right.

A complete network diagram should have one stand point and one finish point. The flow of the diagram should be from left to right. Arrows should not be crossed unless it is completely unavoidable. Arrows should be kept straight and curved or bent. Angle between arrows should as large as possible. Each activity must have a tail or head event. No two or more activities may have same tail and head events. Once the diagram is complete the nodes should be numbered from left to right. It should then be possible to address each activity uniquely by its tail and head event.



Net work diagram example

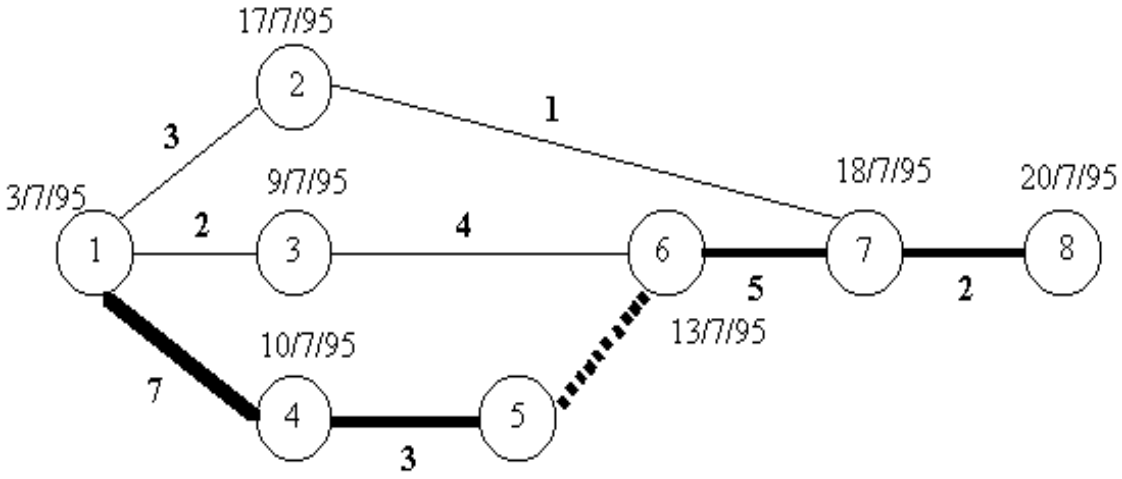


Illustration1

Draw a network diagram from the following activities.

Activity	:1-2	1- 3	1-4	2 - 5	3- 5	4 - 6	5- 6
Time deviation	:2	4	3	1	6	5	7

Solution

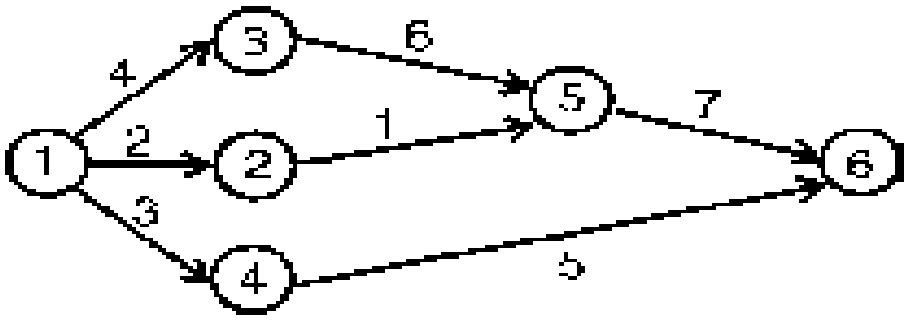
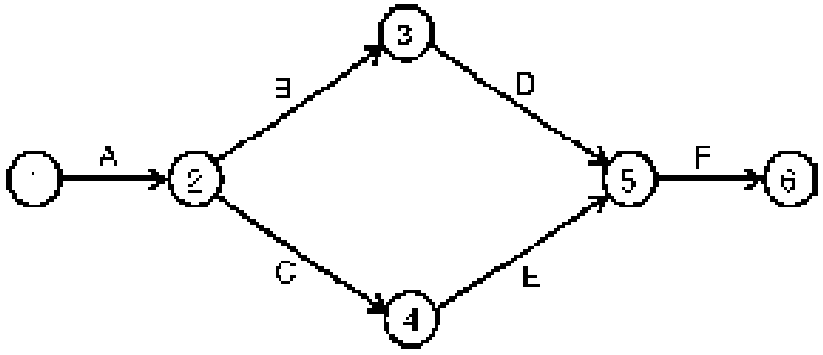


Illustration2

Activity	Description	Predecessor activity
A	Finish component Development	
B	Design marketing programme	A
C	Design production system	A
D	Select Advertising media	B
E	Initial production run	C
F	Release component to market	D, E

Solution



Network techniques

The main network techniques are

1. Critical path Method
2. Program Evaluation Review Technique

Critical path method (CPM)

The Critical Path Method (CPM) is one of several related techniques for doing project planning. CPM is for projects that are made up of a number of individual "activities." If some of the activities require other activities to finish before they

can start, then the project becomes a complex web of activities. CPM provides the following benefits:

- Provides a graphical view of the project.
- Predicts the time required to complete the project.
- Shows which activities are critical to maintaining the schedule and which are not.

Steps in CPM Project Planning

1. Specify the individual activities.
2. Determine the sequence of those activities.
3. Draw a network diagram.
4. Estimate the completion time for each activity.
5. Identify the critical path (longest path through the network)
6. Update the CPM diagram as the project progresses.

Critical path

Those activities which contribute directly to the overall duration of the project constitute critical activities; the critical activities form a chain running through the network which is called critical path. The critical path is the longest path in the network from the starting event to ending event & defines the minimum time required to complete the project. The critical path is denoted by darker or double lines.

Critical event: - The slack of an event is the difference between the latest and earliest events time. The events with zero slack time are called as critical events.

Critical activities: - The difference between latest start time and earliest start time of an activity will indicate amount of time by which the activity can be delayed without affecting the total project duration. The difference is usually called total float. Activities with 0 total float are called as critical activities.

To determine the duration of individual activities, the four activity times are to be computed.

- **Earliest start time: -** The earliest time at which the activity can start given that its precedent activities must be completed first.
- **Earliest finish time: -** It is equal to the earliest start time for the activity plus the time required completing the activity.
- **Latest finish time: -** The latest time at which the activity can be completed without delaying the project.

- **Latest start time: -** It is equal to the latest finish time minus the time required to complete the activity.

Types of float

Float is the amount of time by which completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.

Latest finish time – earliest start time – duration of an activity.

- Free float: - This is concerned commencement of subsequent activity. It may be defined as "time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity.

- Independent Float: - it may be defined as the amount of time by which the start of an activity can be delayed without affecting the earliest start time of any successor activity, assuming that preceding activity has finished at its latest finish time.

Slack time: The *slack time* for an activity is the time between its earliest and latest start time, or between its earliest and latest finish time. Slack is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project.

Computation of EFT and LFT

1. Forward pass

The forward pass method yields the earliest start time and earliest finish times for each activity and indirectly earliest expected occurrence of each event. The computation begins from the start node and move to the end node. To accomplish this, the forward pass computations start with an assumed earliest occurrence time of zero for the initial project event $E_1 = 0$ Earliest start time for activity (I,j) is the earliest event time of the tail end event $ES_{IJ} = E_I$

Earliest finish time of the activity is the earliest start time of the activity plus the duration of the activity. EFij = ES ij+ tij Earliest occurrence time of the event j is the maximum of the earliest finish times of all the activities into that event.

Eg.E_1 =0, $E_2 = E_1$ + activity duration

2. **Backward Pass Method**:

The latest occurrence time specifies the time by which all the activities entering into that event, must be completed without delaying the total project. These are computed by reversing the method of calculation used for earliest

event times. Latest finish time of an activity is equal to the latest vent time j LF_{IJ}= L $_J$. Latest start time of an activity is given by latest completion time minus the activity time. LS_{IJ} = LF_{IJ} - t_{ij} Latest event time for event is the minimum of the latest start time of all activities originating from that event.

Eg.L_{10} =20, L_9 = 20 – activity duration

Illustration 3

A small maintenance project consists of the following Jobs whose precedence relationships are gives below.

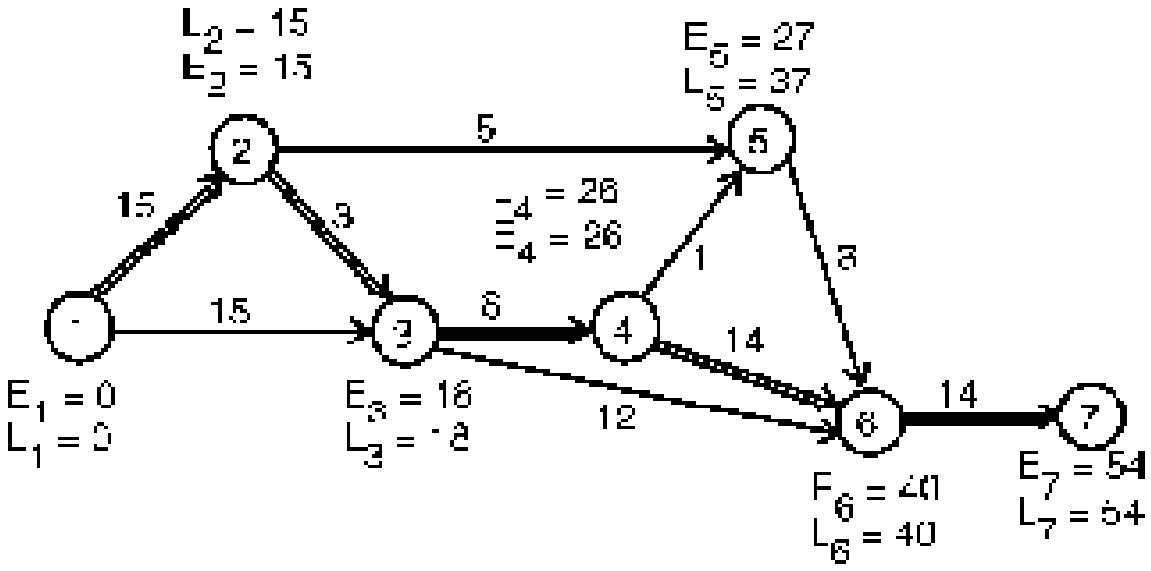
Job : 1-2 1-3 2 -3 2-5 3 -4 3 -6 4 -5 4 - 6 5 – 6 6 -7

Duration : 15 15 3 5 8 12 1 14 3 14

(Days)

1. Draw an arrow diagram
2. Find the total float for each activity
3. Find the critical path and the project duration.

Solution



Forward pass computation

E_1 = 0

E_2 = E_1 + 15 = 0 + 15 = 15

E_3 = Maximum of E_1 + 15 and E_2 + 3

E_1 + 15 = 0 + 15 = 15; E_2 + 3 = 15 + 3 = **18**

E_3 = 18

E_4 = E_3 + 8 = 18 + 8 = **26**

E_5 = Maximum of E_2 + 5, and E_4 + 1

E_2 + 5 = 15 + 5 = 20,

E_4 + 1 = 26 + 1 = **27**

E_5 = **27**

E_6 = Maximum of E_3 + 12, E_4 + 14 and E_5 + 3

E_3 + 12 = 18 + 12 = 30; E_4 + 14 = 26 + 14 = **40**

E_5 + 3 = 27 + 3 = 30

E_6 = **40**

E_7 = E_6 + 14 = 40 + 14 = **54**

Backward pass computation

L_7 = 54

L_6 = L_7 -14 = 54 - 14 = **40**

L_5 = $L_6$3 = 40- 3 = **37**

L_4 = Minimum of L_6 -14 and L_5 -1

L_6 -14 = 40 -14 = 26, L_5 -1 = 37 - 1 = 36

L_4 = **26**

L_3 = Minimum of L_4 - 8 and L_6 -12

L_4-8 = 26 - 8 = 18

L_6-12 = 40 - 12 = 28

L_3 = **18**

L_2 = Minimum of L_5 -5 and L_3 - 3

L_5-5 = 37 - 5 = **32**

L_3-3 = 18 - 3 = **15**

L_2 = **15**

L_1 = Minimum of L_3 -15 and L_2 -15

L_3-15 = 18 -15 = 3, L_2 -15 = 15 -15 = **0**

L_1 = **0**

Critical path is 1 – 2 – 3 – 4 – 6 - 7

The total project completion days = **54**

Activity	Duration	Earliest time		Latest time		Total float
		EST	EFT	LST	LFT	LST-EST
1-2	15	0	15	0	15	0
1-3	15	0	15	3	18	3
2-3	3	15	18	15	18	0
2-5	5	15	20	32	37	17
3-4	8	18	26	18	26	0
3-6	12	18	30	28	40	10
4-5	1	26	27	36	37	10
4-6	14	26	40	26	40	0
5-6	3	27	30	37	40	10
6-7	14	40	54	40	54	0

From the above table we can observe that the critical activities are 1 - 2, 2 - 3, 3 - 4, 4 - 6 and 6 -7. In all these case total float is 0.

The critical path is1 – 2 - 3 - 4 - 6 -7. Project duration is 54.

Programme Evaluation and Review Technique

PERT is designed for scheduling complex projects that involve many inter-related tasks. It improves planning process because: It forms planner to define the projects various components activities. It provides a basis for normal time estimates and yet allows for some measure of optimism or pessimism in estimating the completion dates. It shows the effects of changes to overall plans they contemplated. It provides built in means for ongoing evaluation of the plan. The Program (or Project) Evaluation and Review Technique, commonly abbreviated PERT, is a statistical tool, used in project management, that is designed to analyze and represent the tasks involved in completing a given project.

It shows

- Sequence of tasks
- Which tasks can be performed simultaneously

- Permits determination of the critical path for the individual tasks to be completed on time in order for the project to meet its completion deadline.

Time estimates in PERT

There are three time estimates

1. Optimistic time estimate
2. Most likely time estimate
3. Pessimistic time estimate

1. Optimistic time estimate

This is the fastest time an activity can be completed. For this, the assumption is made that all the necessary resources are available and all predecessor activities are completed as planned. it is that time estimate of an activity when everything is assumed to go as per plan. In other words it is the estimate of minimum possible time which an activity takes in completion under ideal conditions.

Most likely time (m or t_m)

The time which the activity will take most frequently if repeated number of times.

Pessimistic time (b or t):

The unlikely but possible performance time if whatever could go wrong, goes wrong in series. In other words it is the longest time the can take.

From the above time estimates we can calculate the expected time of each activity by using the following formula.

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Illustration 4

A small project is composed of 7 activities, whose time estimates are given below.

Activity	Estimated Duration (weeks)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7

1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

a. Draw a project network and identify all the paths through it.

b. Find the expected duration and variance for each activity. What is the expected project length?

c. Calculate the variance and SD of the project length. What is the probability that the project will be completed?

   i. At least 4 weeks earlier than expected one?

   ii. Not more than 4 weeks later than expected time?

d. If the project due date is 19 weeks, what is the probability of not meeting the due date?

Given Z	:	0.50	0.67	1.00	1.33	2.00
P	:	0.3088	0.2514	0.1587	0.0918	0.0228

(ICWA Final Exam)

Solution

Calculation to find out Expected time and Variance.

Activity	t_o	t_m	t_P	t_e	$\sigma 2$
1 -2	1	1	7	2	1
1- 3	1	4	7	4	1
1- 4	2	2	8	3	1
2- 5	1	1	1	1	0

3-	5	2	5	14	6	4
4-	6	2	5	8	5	1
5-	6	3	6	15	7	4

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\pm^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Forward pass computation

$E_1 = 0$

$E_2 = E_1 + 2 = 0 + 2 = \mathbf{2}$

$E_3 = E_1 + 4 = \mathbf{4}$

$E_4 = E_1 + 3 = \mathbf{3}$

E_5 = Maximum of $E_2 + 1$ and $E_3 + 5$

$E_1 + 1 = 2 + 1 = 3$

$E_3 + 6 = 4 + 6 = 10$

$E_5 = \mathbf{10}$

E_6 = Maximum of $E_5 + 7$ and $E_4 + 5$

$E_5 + 7 = 10 + 7 = 17,$

$E_4 + 5 = 3 + 5 = 8$

$E_6 = \mathbf{17}$

Backward pass computation

$L_6 = 17$

$L_5 = L_6 - 7 = 17 - 7 = 10;$

$L_4 = L_6 - 5 = 17 - 5 = 12$

$L_3 = L_5 - 6 = 10 - 6 = 4;$

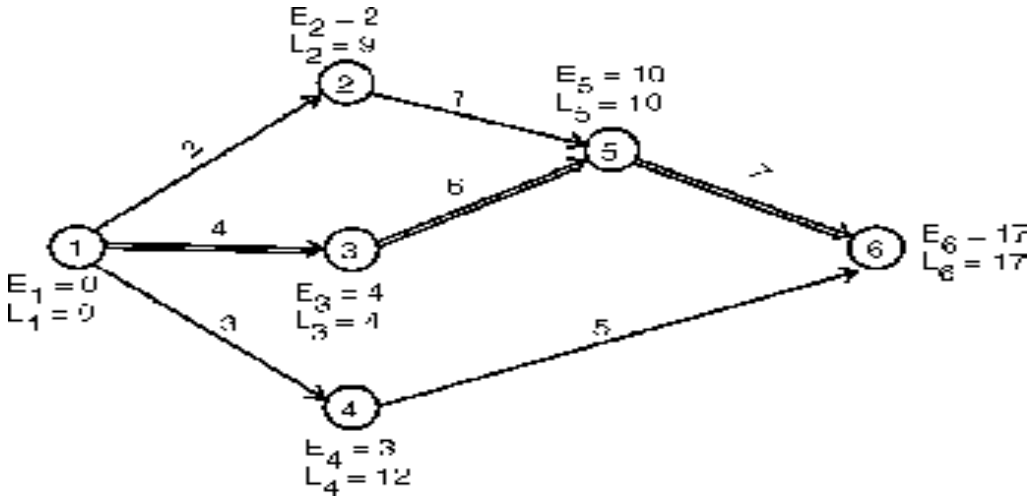
$L_2 = L_5 - 1 = 10\text{-}1 = 9$

L_1 = Minimum of L_2 -2, L_3 -4, L_4 -3

L_2 - 2 = 9 -2 = 7, L_3 - 4 = 4 - 4 = 0,

L_4 - 3 = 12 - 3 = **9**

L_1 = **0**



a. Critical activities are 1-3, 3 - 5, 5 -6

Critical path is 1 - 3 - 5 - 6

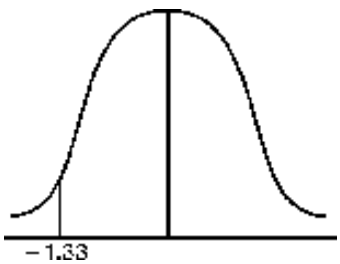
b. Expected project duration length is 17 weeks.

Variance of the critical path is 1 + 4 + 4 = 9 weeks.

$\therefore \pm = \sqrt{9} = 3$

c. (1) Probability that the project will be completed at least 4 weeks earlier than the expected time.

$$Z = \frac{Due\ time - Expected\ time}{\pm}$$

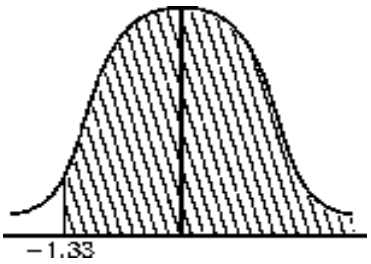


Due time =17 - 4 = **13**

$$Z = \frac{13-17}{3} = \frac{-4}{3} = \mathbf{1.33}$$

Corresponding area value as per statistical table under normal curve is **0.0918**.

Probability that the project will be completed at least 4 weeks earlier = 0.0918100 = **9.18%**



(2) Probability that the project will be completed no more than 4 weeks later than expected time?

$Z = 1.33$

Corresponding area value = 0.0918.

The required area is

= (0.5 - 0.0918) + 0.5

=0.9082

=0.9082100 = **90.82%**

(3) Project due date is 19 weeks, then the probability of not meeting the due date?

$$Z = \frac{19-17}{3} = \frac{2}{3} = 0.67$$

Normal value corresponding to 0.67 is 0.2486

Probability of not meeting the due date the required area is

1 – 0.2486 = 0.7514

That is 0.7514100 = **75.14%**

Difference between PERT & CPM:

PERT is a probability model with uncertainty in activity duration. The duration of each activity is computed from multiple time estimates with a view to take into account time uncertainty. It is applied widely for planning and scheduling research projects. PERT analysis does not usually consider costs. CPM is a deterministic model with well known activity times based upon the past experience. It is used for construction projects and business problems. CPM deals with cost of project schedules and minimization. Under CPM each activity in the network diagram is represented by circle and arrows are used to indicate sequencing requirements. In case of PERT each event is represented by

a circle and arrows are used to indicate activities. In CPM critical path is determined on the basis of float. In PERT critical path is determined on the basis of slack.

Limitations of PERT /CPM:

Limitations of PERT /CPM Network diagrams should have clear starting and ending points, which are independent of each other which may not be possible in real life. Another limitation is that it assumes that manager should focus on critical activities. Resources will be available when needed for completion for an activity is again unreal.

CHAPTER 4

DECISION THEORY

The decision theory is a technique used for decision making in uncertain conditions or situations. It provides a framework and methodology for rational decision making when the outcomes are uncertain. Decision making may be defined as a process which results on the selection from a set of alternative course of actions which is considered to meet the objectives of the decision problem more satisfactory than others judged by decision makers. Decision theory provides a rational framework for choosing between alternative courses of action when the consequences resulting from this choice are imperfectly known. Two streams of thought serve as the foundations: utility theory and the inductive use of probability theory.

Elements common to all decisions

1. Course of action: - There is finite number of decision alternatives available to the decision maker at each point in time when a decision is made. The number and type of such alternatives may depend upon the previous decisions. These alternatives are called course of action.

2. State of nature: - These are future conditions that are not under the control of decision maker. A state of nature may be the state of economy, a weather condition, a political development, etc.

3. Pay off: It is the outcome resulting from each possible combination of alternatives and states of nature. The pay off values is always conditional values because of unknown states of nature.

Decision making under uncertainty

In decision making under pure uncertainty, the decision maker has absolutely no knowledge, not even about the likelihood of occurrence for any state of nature. In such situations, the decision maker's behavior is purely based on his/her attitude toward the unknown.

There are different criteria of decision making under this situation.

1. Optimism criterion

2. Pessimism criterion

3. Equal probabilities

4. Regret criterion.

Optimism (maximax or minimax) criterion

1. As per this criterion the decision maker ensures that he should not miss the opportunity to achieve the largest possible profit. In the maximax criterion the decision maker selects the decision that will result in the maximum of maximum payoffs; an optimistic criterion.

2. **Pessimism (maximin or Minimax) criterion**

   In this criterion the decision maker ensures that he would earn no less than some specified amount. The decision maker selects the alternative that represents the maximum of minima pay off in case of profits. This criterion is also known as Wald's criterion.

3. **Laplace criterion**

   We don't know the states of nature. Hence it is assumed that all states of nature will occur with equal probability.

   Steps

   1. Assign equal probability value to each state of nature.

   2. Compute the expected or average pay off for each alternative by adding all the payoff and dividing by the number of possible states of nature.

      (Probability of state of nature *j) x (pay off value for the combination of alternative i and state of nature j.)*

   3. Select the best expected pay off

Hurwicz criterion

As per this criterion a rational decision maker is neither completely optimistic nor pessimistic. Hurwicz introduced coefficient of optimism. The Hurwicz alpha is a criterion for decision making under complete uncertainty that represents a compromise between the Maximin and Maximax criteria. The alpha is a number between 0 and 1. In the special case where it is one, the criterion reduces to Maximin and in the special case where it is zero the criterion reduces to Maximax. The decision maker can set alpha to a number

between zero and one according to his or her level of optimism.

H (criterion of realism) = $\propto$ (*Maximum in column*) + (1 -$\propto$) (minimum in column)

Select an alternative with best anticipated weighted average pay off value.

Regret criterion or opportunity loss decision criterion or minimax regret decision criterion

Here the decision maker regrets the fact that he adopted a wrong course of action resulting in an opportunity loss of pay off.

Steps

From the given pay off matrix table develop an opportunity pay off matrix.

Find the best pay off. Then deduct other values from the best pay off. For each course of action identify the worst or maximum regret value.

Select the course of action with the smallest anticipated opportunity loss vale.

Illustration 1

A manufacturer manufactures a product of which the principal ingredient is a chemical X. At the moment, the manufacturer spends Rs 1000 per year on supply of X, but there is a possibility that the price may soon increase to four times its present figure because of a worldwide shortage of the chemical. There is another chemical Y, which the manufacture could use in conjunction with a third chemical Z, in order to give the same effect as chemical X. Chemicals Y and Z, would together cost the manufacturer Rs 3,000 per year, but their prices are unlikely to rise. What action should the manufacturer take? Apply the maximin and minimax criteria for decision making and give two set of solutions. If the coefficient of optimism is 0.4 then find the course of action that minimizes cost.

Solution

State of nature	Course of action	
	S_1(use Y and Z)	S_2 (use X)
E_1 (Price of X increases)	- 3000	- 4000
E_2 (Price of X does not increase)	- 3000	- 1000
Column minimum	-3000	- 4000

1. Maxmin criterion - 3000

Hence the manufacturer should adopt S_1 strategy

2. Minimax criterion (opportunity loss)

State of nature	Course of action	
	S_1(use Y and Z)	S_2 (use X)
E_1 (Price of X increases)	0	1000
E_2 (Price of X does not increase)	2000	0
Column minimum	-3000	- 4000
1. Maximum	2000	1000
2. Minimax		1000

Hence the manufacturer should adopt S_2 strategy.

Hurwicz Criterion

The coefficient of optimism is 0.4 so the coefficient of pessimism is **0.6**

Select course of action that optimizes profit or minimizes cost.

Course of action	Best pay off	Worst pay off	H
S_1	- 3000	- 3000	- 3000
S_2	- 1000	- 4000	-2,800

(-3000 ×0.4) + (-3000 ×0.6) = - 3,000

(- 1000×0.4) + (- 4000 ×0.6) = - 2800

Course of action S2 has the least cost (maximum profit). So the manufacturer should adopt this.

Illustration 2

A food products company is planning to introduce a novel product with new packing to replace the existing product at much higher price (S_1) or a moderate change in composition of the existing product with new packaging at a small increase in price (S_2) or a small change in the composition of the existing except the word, 'New' with a negligible increase in the price (S_3). The three possible states of nature of events are (i) high increase in sales (N_1) (ii) no change in sales (N_2) (iii) decrease in sales (N_3). The marketing department of the company worked out the pay offs in terms of yearly new profits for each of the strategies on these events. This is represented in the following table.

	Pay off States of nature		
Strategies	N_1	N_2	N_3
S_1	700	300	150

S_2	500	450	0
S_3	300	300	300

Which strategies should the executive concerned choose on the basis of (a) Maximin criterion

(b) Maximax criterion c) Minimax regret criterion d) Laplace criterion

Solution

	Strategies		
States of nature	S_1	S_2	S_3
N_1	700	500	300
N_2	300	450	300
N_3	150	0	300
Minimum	150	0	300

a) Maximin criterion 300

So we want to select S_3 strategy.

Maximum	700	500	300

b) Maximax 700

So we want to select S_1 strategy.

c) Minimax regret criterion

Opportunity Loss Table

	Strategies			
States of nature	S_1	S_2	S_3	
N_1	0	200	400	(700 -700, 700 – 500 etc)
N_2	150	0	150	
N_3	150	300	0	
Maximum opportunity loss		150	300	450
Minimum of these maximum	150			

Hence we want to select S_1 strategy.

d) Laplace criterion

	Strategies		
States of nature	S_1	S_2	S_3
N_1	700	500	300
N_2	300	450	300
N_3	150	0	300
Average	1150/3	950/3	900/3
	383.3	316.67	300

The average is highest for strategy S_1 . So we may select strategy S_1.

Decision making under risk

It is a probabilistic situation. Here more than one state opt nature exists. The decision maker has sufficient information to assign probability values to the likely occurrence of each of these states. On the basis of knowing the probability distribution of the states of nature, the best decision is to select the course of action which has the largest expected pay off value. There are mainly two methods used to find decisions under risk.

a) Expected Monetary Value

b) Expected opportunity loss criterion

Expected monetary value (EMV)

It is the weighted sum of possible pay off for each alternative.

EMV = conditional profits or losses× corresponding probability of each state of nature.

Illustration 3

For the past 200 days, the sales of cakes (1 kg) from the Lovely Bakery has been as follows.

Daily sales :	0	100	200	300	400
No. of days:	10	60	60	50	20

(a) Calculate the expected sale of cakes.

(b) Production cost per cake (1kg) are Rs 5 and sale price is Rs 10 per cake, and any cake unsold at the end of the day is contracted to a local farmer, who pays Rs 2 per cake. Draw up a pay off table for each sales/production combination.

(c) Compute the expected profit arising from each level of production and determine the optimal policy.

Solution

Calculation of expected demand

a. Daily Demand (x) Probability Expected value

0	10/200 = 0.05	0× 0.05 = 0
100	60/200 = 0.30	100× .30 = 30
200	60/200= 0.30	200 ×.30 = 60
300	50/200 = 0.25	300 ×0.25 = 75
400	20/200= 0.10	400 ×0.10 = 40
		205

Total number of days = 10 + 60 + 60 + 50 + 20 = 200

The expected demand is 205 cakes

(b)Profit of sale of cake = 10 - 5 = Rs 5

Loss on unsold cake = 5 - 2 = Rs 3

Pay off Matrix Table

Dem.	Prob.	Course of Action							
	1	0	100	P Pr	200	P×Pr	300	P×Pr	400	P×Pr
0	0.5	0	300	15	600	30	900	45	1200	60
100	0.30	0	500	150	200	60	100	30	400	120
200	0.30	0	500	150	1000	300	700	210	400	120
300	0.25	0	500	125	1000	250	1500	375	1200	300
400	0.10	0	500	50	1000	100	1500	150	2000	200
Expected				0		46	680		660	440

pay off

P = profit, Pr = probability

When course of action is 100, Demand is 0, and then loss of unsold stock is 100 × 3 = 300

When course of action is 100 and demand is 100, then profit is 100 × 5 = 500.

When course of action is 200, Demand 0, then loss is 200 × 3 = 600

When course of action is 200, demand 100, then profit is 500 - 300 (loss o unsold stock) = 200.

When course of action is 300, demand is 100,

Profit (100× 5) = 500, Loss on unsold stock = 200 × 3 = 600, Expected loss = 600 ×500 = 100

When course of action is 300, demand 200.

Profit = 200 × 5 = 1000, loss of unsold stock = 100× 3 = 300

Expected profit = 1000× 300 = 700

(c) From the table we can see that the maximum expected pay off is 680. So the optimal policy is to produce 200 units.

Expected Value of perfect information

If the decision maker is able to acquire perfect information about the occurrence of various states of nature, then he will be able to select a course of action that yields the desired pay off for whatever state of nature actually happens.

The expected value of perfect information is the expected outcome with perfect information.

EVPI = Expected value with perfect information – maximum EMV.

Expected value with perfect information = (best outcome for consequence for first state of nature ×probability of first state of nature) + best outcome for consequence for second state of nature ×probability of second state of nature)……

Expected value of regrets

The decision is to select the strategy with minimum expected opportunity loss.

Illustration4

A retailer purchases cherries every morning at Rs 50 a case and sells them for Rs 80 a case. Any case that remains unsold at the end of the day can be disposed of the next day at a salvage value of Rs 20 per case. (Thereafter they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of the sales for the past 120 days.

Cases sold :	15	16	17	18
Number of days:	12	24	48	36

Find out how many cases should the retailer purchase per day in order to maximize his profit?

Solution

Demand	Prob	Conditional profit				Expected pay off			
		15	16	17	18	15	16	17	18
15	0.1	450	420	390	360	45	42	39	36
16	0.2	450	480	450	420	90	96	90	84
17	0.4	450	480	510	480	180	192	204	192
18	0.3	450	480	510	540	135	144	153	162
EMV		450	474	486	474				

The highest EMV of Rs 486. It corresponds to the course of action 17. Hence the retailer should purchase 17 cases of cherries every morning.

Illustration5

A television dealer finds that the cost of a TV in stock for a week is Rs. 30 and the cost of a unit shortage is Rs. 70. For one particular model of television the probability distribution of weekly sales is as follows.

Weekly Sales:	0	1	2	3	4	5	6
Probability:	0.10	0.10	0.20	0.25	0.15	0.15	0.05

How many units per week should the dealer order? Also, find EVPI? (CA Exam)

Solution

Sales units	Prob.	0	$A_1 \times p$	1	$A_2 \times p$	2	$A_3 \times p$	3	$A_4 \times p$	4	$A_5 \times p$	5	$A_6 \times p$	6	$A_7 \times p$
		A_1		A_2		A_3		A_4		A_5		A_6		A_7	
0	0.10	0	0	30	3	60	6	90	9	120	12	150	15	180	18
1	0.10	70	7	30	3	60	6	90	9	120	12	150	15	180	18
2	0.20	140	28	100	20	60	12	90	18	120	24	150	30	180	36
3	0.25	210	52.5	170	42.5	130	32.5	90	22.5	120	30	150	37.5	180	45

4	0.15	280	42	240	36	200	30	160	24	120	18	150	22.5	180	27
5	0.15	350	52. 5	310	46.5	270	40.5	230	34.5	190	28.5	150	22.5	180	27
6	0.05	420	21	380	19	340	17	300	15	260	13	220	11	180	9
Expected cost			203.0		170		144		132		137.5		153.5		180

The minimum expected cost is 132. So we want to stock 3 units every week.

EVPI = Expected value with perfect information - Maximum EMV.

EMV with P1 = 0.10 × 0 + 0.10 × 30 + 0.20× 60 + 0.25× 90 + 0.15 × 120 + 0.15× 150 + 0.05 × 180 = 87

EVPI = **Rs. 45**

Decision making under competition or conflict

Game theory

Game theory is a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory is a branch of mathematics that deals with the analysis of games (i.e., situations involving parties with conflicting interests). In addition to the mathematical elegance and complete "solution" which is possible for simple games, the principles of game theory also find applications to complicated games such as cards, checkers, and chess, as well as real-world problems as diverse as economics, property division, politics, and warfare. It is concerned with decision making in organizations where the outcome depends upon decisions of two or autonomous players**,** one of which may be nature itself and where no single decision maker has full control over the outcomes. It aims to find optimal solutions to situations of conflict and cooperation under the assumption that players are instrumentally rational and act in their best interests.

The basic constituents of any game are its participating, autonomous decision makers called players. A game must have two players. The total number of players may be large, but must be finite, and must be known. Each player must have more than one choice.

Decision trees

A **decision tree** is a decision support tool that uses a tree-like graph or model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility. It is one way to display an algorithm. Decision trees are commonly used in operations research, specifically in decision analysis, to help identify a strategy most likely to reach a goal. It is a

schematic tree-shaped diagram used to determine a course of action or show a statistical probability. Each branch of the decision tree represents a possible decision or occurrence. The tree structure shows how one choice leads to the next, and the use of branches indicates that each option is mutually exclusive.

A decision tree consists of 3 types of nodes:

1. Decision nodes - commonly represented by squares
2. Chance nodes - represented by circles
3. End nodes - represented by triangles

Example

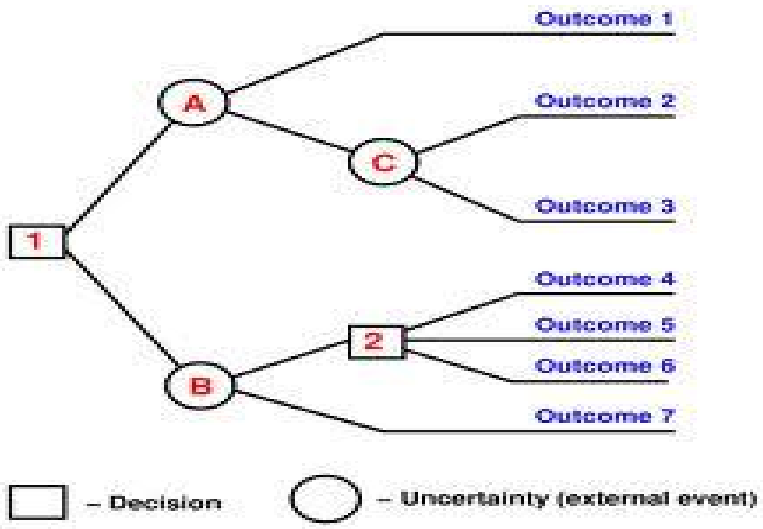


Illustration 6

There is 40% chance that a patient admitted to the hospital, is suffering from cancer. A doctor has to decide whether a serious operation should be performed or not. If the patient is suffering from cancer and the serious operation is performed, the chance that he will recover is 70%, otherwise it is 35%. On the other hand, if the patient is not suffering from cancer and the serious operation is performed the chance that he will recover is 20%, otherwise it is 100%. Assume that recovery and death are the only possible results. Construct an appropriate decision tree. What decision should the doctor take?
(ICWA (new) June 1995)

Solution

Chance of suffering from cancer = 40%

Chances of not suffering from cancer = 60%

Patients in suffering from cancer, the patient will recover after the serious operation = 70%

The patient will not recover after operation = 30%

Probability that the patient will recover after serious operation = 40×70/100 = 0.28

Probability that the patient will not recover after serious operation = 40×30/100 = 0.12

Probability that the patient will recover without serious operation = 40 ×35/100 = 0.14

Probability that the patient will not recover without serious operation = 40×65/100 = 0.26

Patient is not suffering from cancer.

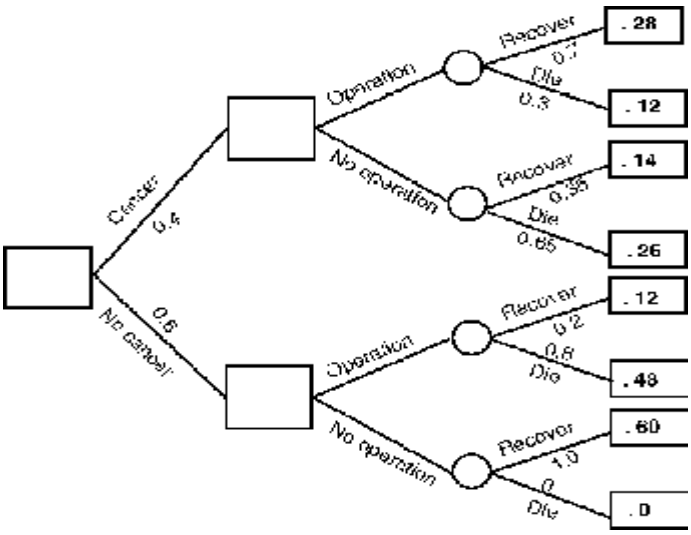
Probability that the patient will recover with serious operation = 60×20/100 = 0.12

Probability that the patient will not recover with serious operation = 60 ×80/100 = 0.48

Probability that the patient will recover without operation = 60× 100/100 = 0.60

Probability that the patient will not recover without operation = .0

Decision Tree Diagram



CHAPTER 5

TRANSPORTATION MODEL

The transportation problem involves determining a minimum-cost plan for shipping from multiple sources to multiple destinations. A transportation model is used to determine how to distribute supplies to various destinations while minimizing total shipping cost. In this case, a shipping plan is produced and is not changed unless factors such as supply, demand, or unit shipping costs change. The variables in this model have a linear relationship and therefore, can be put into a transportation table. The table will have a list of origins and each one's capacity or supply quantity period. It will also show a list of destinations and their respective demands per period. Also, it will show the unit cost of shipping goods from each origin to each destination. The transportation model is a valuable tool in analyzing and modifying existing transportation systems or the implementation of new ones. In addition, the model is effective in determining resource allocation in existing business structures. The model requires a few keys pieces of information, which include the following:

Origin of the supply

Destination of the supply

Unit cost to ship

The transportation model can also be used as a comparative tool providing business decision makers with the information they need to properly balance cost and supply. The use of this model for capacity planning is similar to the models used by engineers in the planning of waterways and highways.

The basic idea in a transportation problem is that there are sites or sources of product that need to be shipped to destinations. Typically the routes and the amounts shipped on each route must be determined and the goal is to minimize the cost of shipping. The constraints are that you cannot ship more from a source than made at that source and you do not want to ship more to a place than needed. The main objective of a transportation problem is to satisfy the demand destination requirements within the plant capacity constraints at minimum transportation cost.

Source is supply. Factory capacity is supply. Destination is demand. Warehouse requirements can also be called demand.

Solution to a transportation problem

The following are the steps to solve a transportation problem.

1. Define the objective function. That is minimization of cost.
2. Set up a transportation table.
3. Develop an initial feasible solution.
4. Verify the initial feasible solution is optimum or not.
5. If the solution is not optimal, then modify the allocation.
6. Repeat step and six till we get an optimal solution.

Methods to find out initial feasible solution

There are mainly three methods used to find out initial feasible solution.

1. North West corner method (NWCM)
2. Least cost method (LCM)
3. Vogel's approximation method

North West corner method

It is a simple method to obtain an initial solution. This method does not take into account the cost of transportation on any route of transportation.
Steps

1. Start with the cell at the North West corner of the transportation matrix and allocate commodity equal to the minimum of the rim values for the first row and first column.

2. If allocation done in step I is equal to the supply available at first source, then move vertically down to the cell (2,1) in the second row and first column.

3. Apply step one again for next allocation.

4. If allocation made in step 2 is equal to the supply available at second source, then move vertically down to the cell (3,1).

5. Continue the procedure till an allocation is made in the south east corner cell of the transportations table.

Illustration 1

For the following transportation problem, obtain initial feasible solution by
(a) North West Corner method (2) Least cost method (3) Vogel's Approximation method.

Origin	Destination				Availability
	1	2	3	4	
1	10	8	11	7	20
2	9	12	14	6	40
3	8	9	12	10	35
Requirement	16	18	31	30	95

Solution

In this problem total availability is equal to requirement. So it has an initial basic feasible solution.

(1) North West Corner Rule

Destination

Origin	1	2	3	4	Availability
1	10	8	11	7	20
2	9 (16)	12 (4)	14	6	40
3	8 (14)	9 (26)	12	10	35
Required	16	18	31 (5)	30 (30)	95

The entries made in the table is given below

1. Minimum of 16, and 20 is assigned in (1, 1). Column 1 is exhausted and eliminated.

   The balance in availability corresponding to row is 4.

2. Minimum of 4 and 18 is allocated to cell (1, 2). Row '1' is exhausted and deleted. The balance in requirement (column 2) is (18 - 4) = 14.

3. The minimum of 14 and 40 is allocated to cell (2, 2). Column 2 is exhausted and deleted.

   The balance in availability row is 26.

4. The minimum of 26 and 31 is allocated to cell. (2, 3). So row 2 is exhausted and deleted.

   The balance in requirement is (31 - 26) = 5

5. The minimum of 5 and 31 is allocated to cell (3, 3). So column 3 is exhausted and deleted.

   The balance in availability row is (35 - 5) = 30

6. 30 is allocated to cell (3, 4)

   The transportation cost $= 10 \times 16 + 8 \times 4 + 12 \times 14 + 26 \times 14 + 12 \times 5 + 10 \times 30$

   = Rs. **1084**

Least cost method

Steps

1. Select the cell with the lowest transportation cost among all the rows or columns of the transportation table,
2. If there are more than two lowest same cost, we can select the cell for allocation arbitrarily, among the lowest cost cells.
3. Assign maximum units in the lowest cost cell. Then we can exhaust either a row total or a column on the basis of allocation. Eliminate that column or row.
4. Consider the reduced matrix table and select another lowest cost cell. Then allocate the maximum units in that cell. On the basis of that we can exhaust either a row or column.
5. Continue the process till all the available quantities are exhausted.

Initial solution - Least cost method (illustration1)

	1	2	3	4	Availability
1	10 (18)	8	11 (2)	7	20

2		9	12	10 14	30 6	40
3	16	8	9	19 12	10	35
Required	16	18	31	30	95	

The table indicates that $\sum a_1 = \sum b_1 = 95$. So there exists an initial feasible solution.

1. The lowest cost in table is 6. So the minimum of 40, 30 is allocated to cell (2, 4). The column 4 is exhausted and deleted. The balance in 'availability' is 40 – 30 = 10.
2. The next lowest cost is 8. Cell (1, 2) is selected arbitarily. The minimum of 18 and 20 that is 18 is allocated on this cell. The balance in 'availability' is 20 – 18 = 2. Column 2 is exhausted and deleted.
3. The next lowest cost is the other '8' in cell (1, 3). The minimum of 16 and 35 is allocated to cell (1, 3). Column 1 is exhausted and deleted. The balance in availability is 35 – 16 = 19
4. The next minimum cost is 11. Minimum of 2 (see point 2) and 31 is allocated to cell (1, 3). So row '1' is exhausted and deleted. The balance in requirementis 31 – 2 = 29.
5. The next minimum cost is 12. So select the cell (3, 3). Minimum of 29 and 19 is allocated in that cell. So row '3' is exhausted and eliminated.
6. The next cost is 14. The minimum of 10, 10 is to be allocated to cell (2, 3).

The transportation cost $= 8 \times 16 + 8 \times 18 + 2 \times 11 + 14 \times 10 + 12 \times 19 + 6 \times 30$

= Rs. **842**

3. Vogel's Approximation Method

Vogel's method is based on the concept of opportunity cost. Opportunity cost is the cost incurred for forgone opportunities or penalties.

Steps

1. Find the penalty cost namely the difference between the smallest and next smallest costs in each row and column.
2. Among the penalties found in step (1), select the maximum penalty. If there is a tie relate with maximum penalties, select any one arbitarily.
3. From the selected column or row (as per step 2) find out the cell which is having lowest cost. As much as quantity must be allocated to this cell by co nsidering the demand and supply.

4. The column or row which is exhausted, is to be deleted.

The above steps is to be continued till an initial feasible solution is attained.

It should be noted that if column is exhausted, then there will be a change in row penalty and vice versa.

(Illustration1)

1. The least cost in row '1' is 7 and 8. So the penalty (P_1) is 8 – 7 = **1**. In the second row P_1 is 9 – 6 = **3**. In 3rd row P_1 is 9 – 8= **1**.

Column wise penalty

In the first column P_1 = 9 – 8 = **1**

In the second column P_1 = 9 – 8 = **1**

In the 3rd column, P_1 = 12 – 11 = **1**

In the 4th column P_1 = 7 – 6 = **1**

Allocation 1

	1	2	3	4	Availability	P_1
1_1	10	8	11	7	20	1
2_2	9	12	14	6 (30)	40	3
3_3	8	9	12	10	35	1
Requirement	16	18	31	30	95	
P_1	1	1	1	1		

There fore the maximum penalty among column and row is 3. The least cost corresponding P_1 (3) is 6. So assign 30 to cell (2, 4). The balance in availability column is 10. Column '4' is exhausted and deleted.

Allocation 2

	1	2	3	Availability	P_1
1	10	8	11	20	2
2	9 (10)	12	14	10	3
3	8	9	12	35	1
Requirement	16	18	31	65	

P_2	1	1	1	1

As per the table maximum penalty is 3. The lowest cost in row 2 is 9. So '10' is allocated in cell (2, 1). Row 2 is exhausted and deleted. The balance in corresponding requirement is 6.

				Allocation 3	
	1	2	3	Availability	P_3
1	10	8	11	20	2
3	(6) 8	9	12	35	1
Requirement	6	18	31		
P_3	2	1	1		

There are two maximum penalties - as '2'. Select the Allocation arbitarily. The minimum cost in column 1 is 8. So allocate '6' to cell (3, 1). The balance in corresponding availability is (35 - 6) = **29**. Column 1 is exhausted and deleted.

			Allocation 4	
	2	3	Availability	P_4
1	(18) 8	11	20	3
3	9	12	29	3
Requirement	18	31		
P_3	1	1		

The maximum penalty is 3. Select the first row aribitarily. The minimum cost in that row is '8' Allocate 18 in the cell (1, 2). The balance in the corresponding availability is 20 - 18 = 2. Column 2 is exhausted and deleted.

		Allocation 5
	3	Availability
1	11	2
3	(29) 12	29
Requirement	31	

Various allocation done to the cells are given below.

Origin	Designation				Availability
	1	2	3	4	
1	10	8 (18)	11 (2)	7	20
2	(10) 9	12	14	(30) 6	40
3	(6) 8	9	12 (29)	10	35
Required	16	18	31	30	95

The transportation cost = $9 \times 10 + 8 \times 6 + 8 \times 18 \times 11 \times 2 + 12 \times 29 + 6 \times 30$ = **Rs. 832**