

18/6/18;

UNIT-1

Quadratic Equations

- * A Quadratic Equation in the variable 'x' is in the form of $ax^2 + bx + c = 0$
- * It is also called as the general form of Quadratic Equation or the standard form of Quadratic Equation
- * Where, a, b, c are any real number and $a \neq 0$ but if $b = 0$ then it becomes $bx + c$ which is not a Quadratic Equation but only a simple linear Equation.
- * If $b = 0$ then it becomes $ax^2 + c = 0$ which is also called as Pure Quadratic Equation Eg: $x^2 - 2x - 3 = 0$

* Roots of the Quadratic Equation:

Let α, β be the roots of Quadratic Equations such that:

→ Sum of the roots $= \alpha + \beta = -b/a$

→ Product of the roots $= \alpha\beta = c/a$

→ Hence the Quadratic Equation is $x^2 - (\text{sum})x + \text{product} = 0$

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$x^2 - (-b/a)x + c/a = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$\frac{ax^2 + bx + c}{a} = 0$$

$$\Rightarrow ax^2 + bx + c = 0$$

* Nature of the roots:

→ Discriminant $\Rightarrow b^2 - 4ac = \Delta$

Case 1: If $b^2 - 4ac > 0$, then the roots are real & distinct or different.

Case 2: If $b^2 - 4ac$ is a Perfect square then the roots are real and unequal

Case 3: If $b^2 - 4ac = 0$ then the roots are real & equal and equal to $-b/2a$

Case 4: If $b^2 - 4ac < 0$, then the roots are imaginary i.e. no roots

* Note: To find the root with the help of formula first find $b^2 - 4ac$

* Methods of Solving a Quadratic Equation:

→ There are 3 methods of solving the Quadratic equation

- 1) Method of factorisation
- 2) Method of completing the squares
- 3) Method of application of formula

$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

* Method 1:

$$1) 4x^2 - 25 = 0 \Rightarrow 4x^2 = 25 \Rightarrow x^2 = \frac{25}{4} \Rightarrow x = \pm \frac{5}{2}$$

* Formula:

$$1) (a+b)^2 = a^2 + 2ab + b^2$$

$$2) a^2 + b^2 = (a+b)^2 - 2ab$$

$$3) (a-b)^2 = a^2 - 2ab + b^2$$

$$4) a^2 + b^2 = (a-b)^2 + 2ab$$

$$5) (a^2 - b^2) = (a+b)(a-b)$$

$$6) (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$7) (a+b)^2 - (a-b)^2 = 4ab$$

$$8) (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$$

$$9) (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$$

$$10) a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (a+b)(a^2 - ab + b^2)$$

$$11) a^3 - b^3 = (a-b)^3 + 3ab(a-b) = (a-b)(a^2 + ab + b^2)$$

$$* \sqrt[4]{4x^7 - 25} = 0$$

$$2^7 \cdot x^7 - 5^7 = 0$$

$$(2x)^7 - 5^7 = 0$$

$$\text{WKT } (a^7 - b^7) = (a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$$

$$2x^7 - 5^7 = (2x-5)(2x^6 + 2x^5 \cdot 5 + 2x^4 \cdot 5^2 + 2x^3 \cdot 5^3 + 2x^2 \cdot 5^4 + 2x \cdot 5^5 + 5^6) = 0$$

$$\Rightarrow 2x-5=0$$

$$x = 5/2$$

$$2x+5=0$$

$$x = -5/2$$

$$* \sqrt[5]{5x^7 - 121} = 0$$

$$5x^7 = 121$$

$$x^7 = \frac{121}{5}$$

$$x = \pm \sqrt[7]{\frac{121}{5}}$$

$$* \sqrt[5]{5x^7 - 125} = 0$$

$$5x^7 = 125$$

$$x^7 = \frac{125 \cdot 25}{5}$$

$$x^7 = 25$$

$$x = 5$$

16/18

$$* x^2 - 5x + 6 = 0$$

$$\text{Sol } x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \text{ or } 2$$

$$* x^2 - 6x + 8 = 0$$

$$x^2 - 2x - 4x + 8 = 0$$

$$x(x-2) - 4(x-2) = 0$$

$$(x-2)(x-4) = 0$$

$$x = 2 \text{ or } 4$$

$$* 2x^2 - 5x + 3 = 0$$

$$2x^2 - 6x + x + 3 = 0$$

$$2x(x-3) + 1(x-3) = 0$$

$$(x-3)(2x+1) = 0$$

$$x = 3 \text{ or } -1/2$$

$$* 2x^2 - 5x + 3 = 0$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(x-1)(2x-3) = 0$$

$$x = 1 \text{ or } 3/2$$

$$* x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0$$

$$x(x-5) + 2(x-5) = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5 \text{ or } -2$$

$$* 2x^2 + x - 6 = 0$$

$$2x^2 + 4x - 3x - 6 = 0$$

$$2x(x+2) - 3(x+2) = 0$$

$$(x+2)(2x-3) = 0$$

$$x = -2 \text{ or } 3/2$$

$$* 3x^2 - 5x + 2 = 0$$

$$3x^2 - 2x - 3x + 2 = 0$$

$$x(3x-2) - 1(3x-2) = 0$$

$$(x-1)(3x-2) = 0$$

$$x = 2/3 \text{ or } 1$$

$$* 100x^2 - 20x + 1 = 0$$

$$100x^2 - 10x - 10x + 1 = 0$$

$$10x(10x-1) - 1(10x-1) = 0$$

$$(10x-1)(10x-1) = 0$$

$$x = 1/10, 1/10$$

$$* x^2 - 20x + 100 = 0$$

$$x^2 - 10x - 10x + 100 = 0$$

$$x(x-10) - 10(x-10) = 0$$

$$(x-10)(x-10) = 0$$

$$x = 10, 10$$

$$* 16x^2 - 8x + 1 = 0$$

$$16x^2 - 8x + 1 = 0$$

$$16x^2 - 4x - 4x + 1 = 0$$

$$4x(4x-1) - 1(4x-1) = 0$$

$$(4x-1)(4x-1) = 0$$

$$x = 1/4, 1/4$$

$$* x(x+4) = 12$$

$$x^2 + 4x - 12 = 0$$

$$x^2 + 6x - 2x - 12 = 0$$

$$x(x+6) - 2(x+6) = 0$$

$$(x+6)(x-2) = 0$$

$$x = -6 \text{ or } 2$$

$$* \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$x(\sqrt{2}x+5) + \sqrt{2}(\sqrt{2}x+5) = 0$$

$$(\sqrt{2}x+5)(x+\sqrt{2}) = 0$$

$$x = -5/\sqrt{2}, -\sqrt{2}$$

$$* \sqrt{3}x^2 + 7x + 4\sqrt{3} = 0$$

$$\sqrt{3}x^2 + 4x + 3x + 4\sqrt{3} = 0$$

$$x(\sqrt{3}x+4) + \sqrt{3}(\sqrt{3}x+4) = 0$$

$$(\sqrt{3}x+4)(x+\sqrt{3}) = 0$$

$$x = -4/\sqrt{3}, -\sqrt{3}$$

$$* x^2 - 3/2x = 2$$

$$x^2 - 3/2x - 2 = 0$$

$$\frac{2x^2 - 3x - 4}{2} = 0$$

$$2x^2 - 3x - 4 = 0$$

continued with formula

$$* 3(x-4)^2 - 5(x-4) = 12$$

$$\text{WKT } (a-b)^2 = a^2 - 2ab + b^2$$

$$3(x^2 - 8x + 16) - 5x + 20 - 12 = 0$$

$$3x^2 - 24x + 48 - 5x + 8 = 0$$

$$3x^2 - 29x + 56 = 0$$

$$3x^2 - 21x - 8x + 56 = 0$$

$$3x(x-7) - 8(x-7) = 0$$

$$(x-7)(3x-8) = 0$$

$$x = 7 \text{ or } 8/3$$

3.6/14
* Another Method

$$3(x-4)^2 - 5(x-4) - 12 = 0$$

$$\text{let } x-4 = a$$

$$3a^2 - 5a - 12 = 0$$

$$3a^2 - 9a + 4a - 12 = 0$$

$$3a(a-3) + 4(a-3) = 0$$

$$(a-3)(3a+4) = 0$$

$$19) 2(x+2)^2 - 5(x+2) + 3 = 0$$

$$2(x^2 + 4x + 4) - 5x - 10 + 3 = 0$$

$$2x^2 + 8x + 8 - 5x - 10 + 3 = 0$$

$$2x^2 + 3x + 1 = 0$$

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x+1) + 1(x+1) = 0$$

$$(x+1)(2x+1) = 0$$

$$x = -1 \text{ or } -1/2$$

$$21) \frac{x}{a} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$$

$$\frac{x^2 + b^2}{bx} = \frac{a^2 + b^2}{ab}$$

$$(x^2 + b^2)ab = (a^2 + b^2)bx$$

$$abx^2 + ab^3 = a^3x + ab^2x + b^3x$$

$$abx^2 + ab^3 - a^3x - ab^2x - b^3x = 0$$

$$ab(x^2 - bx) - b^3x = 0$$

$$ab(x^2 + b^2) - bx^2$$

Case I

$$a-b = a = x-4$$

$$x-4 = 3$$

$$x = 7$$

Case II

$$x-4 = -4/3$$

$$x = -4/3 + \frac{4 \times 3}{3}$$

$$= -4/3 + 12/3 = \frac{-4+12}{3} = 8/3$$

7

$$20) \frac{9x-2}{3} + \frac{4x^2-7}{4x^2+3} = \frac{6x-1}{2}$$

$$\frac{(9x-2)(4x^2+3) + 3(4x^2-7)}{3(4x^2+3)} = \frac{6x-1}{2}$$

$$\frac{36x^2 + 27x - 8x^2 - 6 + 12x^2 - 21}{3(4x^2+3)} = \frac{6x-1}{2}$$

$$72x^2 + 54x - 16x^2 - 12 + 24x^2 - 42 = (6x-1)(12x^2+9)$$

$$72x^2 + 54x - 16x^2 - 12 + 24x^2 - 42 = 72x^3 + 54x - 16x^2 - 12 + 24x^2 - 42$$

$$\Rightarrow 72x^3 + 54x - 16x^2 - 12 + 24x^2 - 42 = 72x^3 + 54x - 16x^2 - 12 + 24x^2 - 42$$

$$\Rightarrow 72x^3 - 9x^2 - 20x^2 - 45 = 0$$

$$x = 45/20 = 9/4 \Rightarrow x = \pm \sqrt{9/4}$$

$$x = \pm 3/2$$

8

$$21) \frac{x}{a} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$$

$$\frac{x^2 + b^2}{bx} = \frac{a^2 + b^2}{ab}$$

$$ax^2 + b^2a = a^3x + b^3x$$

$$ax^2 + ab^2 - a^3x - b^3x = 0$$

$$ax^2 - a^3x - b^3x + ab^2 = 0$$

$$ax(x-a) - b^3(x-a) = 0$$

$$(x-a)(ax-b^3) = 0$$

$$\begin{array}{l|l} x-a=0 & ax-b^3=0 \\ x=a & ax=b^3 \\ & x=b^3/a \end{array}$$

$$22) \sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$$

$$\sqrt{3}x^2 + 12x - 2x - 8\sqrt{3} = 0$$

$$\sqrt{3}x^2 + 3 \cdot 4x - 2x - 8\sqrt{3} = 0$$

$$\sqrt{3}x(x + 4\sqrt{3}) - 2(x + 4\sqrt{3}) = 0$$

$$(x + 4\sqrt{3})(\sqrt{3}x - 2) = 0$$

$$x = -4\sqrt{3}, 2/\sqrt{3}$$

21/6/18

$$24) y^2 - 32y + 32 = 0$$

$$y^2 - 32y - y + 32 = 0$$

$$y(y - 32) - 1(y - 32) = 0$$

$$(y - 32)(y - 1) = 0$$

$$y = 32, 1$$

$$23) 3(x-1)^2 - 5(x-1) + 2 = 0$$

$$3(x^2 - 2x + 1) - 5x + 5 + 2 = 0$$

$$3x^2 - 6x + 3 - 5x + 7 = 0$$

$$3x^2 - 11x + 10 = 0$$

$$3x^2 - 6x - 5x + 10 = 0$$

$$3x(x-2) - 5(x-2) = 0$$

$$(x-2)(3x-5) = 0$$

$$x = 2, 5/3$$

9

$$25) (2x+3)(2x+5)(x-1)(x-2) = 30$$

$$[(2x+3)(x-1)][(2x+5)(x-2)] = 30$$

$$[2x^2 - 2x + 3x - 3][2x^2 - 4x + 5x - 10] = 30$$

$$[2x^2 + x - 3][2x^2 + x - 10] = 30$$

$$\text{let } a = 2x^2 + x$$

$$(a-3)(a-10) = 30$$

$$a^2 - 10a - 3a + 30 = 30$$

$$a^2 - 13a = 0$$

$$a(a-13) = 0$$

$$a = 0 \mid a = 13$$

Case 1: $a = 0$

$$2x^2 + x = 0$$

$$x(2x+1) = 0$$

$$x = 0 \mid 2x+1 = 0$$

$$x = -1/2$$

Case 2:

$$2x^2 + x = 13$$

$$2x^2 + x - 13 = 0$$

$$26) (2x-7)(x^2-9)(2x+5)=91$$

$$\text{WKT } (a^2-b^2)=(a-b)(a+b)$$

$$\Rightarrow (2x-7)(x+3)(x-3)(2x+5)=91$$

$$\Rightarrow (2x^2-6x-7x+21)(2x^2+5x+6x+15)=0$$

$$\Rightarrow (2x^2-13x+21)(2x^2+11x+15)=0$$

$$\Rightarrow (2x-7)(x-3)(x+3)(2x+5)=91$$

$$\Rightarrow (2x^2-6x-7x-21)(2x^2+5x+6x+15)=91$$

$$\Rightarrow (2x^2-x-21)(2x^2-x-15)=91$$

$$\text{let } 2x^2-x=a$$

$$(a-21)(a-15)=91$$

$$a^2-15a-21a+315=91$$

$$a^2-15a-21a+224=0$$

$$a^2-36a+224=0$$

$$\Rightarrow a^2-28a-8a+224=0$$

$$\Rightarrow a(a-28)-8(a-28)=0$$

$$(a-28)(a-8)=0$$

$$2x^2-x=98$$

$$2x^2-x-28=0$$

$$2x^2-8x+7x-28=0$$

$$2x(x-4)+7(x-4)=0$$

$$(2x+7)(x-4)=0$$

$$x=-7/2, x=4$$

$$2x^2-x=8$$

$$2x^2-x-8=0$$

$$2x^2$$

$$27) (x^2-3x)^2-8(x^2-3x)-20=0$$

$$\text{let } x^2-3x=a$$

$$a^2-8a-20=0$$

$$a^2-10a+2a-20=0$$

$$a(a-10)+2(a-10)=0$$

$$(a-10)(a+2)=0$$

$$a=-2 \quad | \quad a=10$$

$$x^2-3x=-2$$

$$x^2-3x+2=0$$

$$x^2-2x-x+2=0$$

$$x(x-2)-1(x-2)=0$$

$$(x-2)(x-1)=0$$

$$x=2 \text{ or } 1$$

$$x^2-3x=10$$

$$x^2-3x-10=0$$

$$x^2-5x+2x-10=0$$

$$x(x-5)+2(x-5)=0$$

$$(x-5)(x+2)=0$$

$$x=5, -2$$

Roots are 2, 1, 5, -2.

$$28) \sqrt{1-5x} + \sqrt{1-3x} = 2$$

$$\Rightarrow \sqrt{1-5x} = 2 - \sqrt{1-3x}$$

SOBS

$$(1-\sqrt{5})(\sqrt{1-5x})^2 = (2-\sqrt{1-3x})^2$$

$$1-5x = 2^2 - 2 \cdot 2 \cdot \sqrt{1-3x} + \sqrt{1-3x}^2$$

$$1-5x = 4 - 4\sqrt{1-3x} + 1-3x$$

$$1-5x-4-1+3x = -4\sqrt{1-3x}$$

$$-4-2x = -4\sqrt{1-3x}$$

SOBS

$$(-4-2x)^2 = (-4)^2 \sqrt{1-3x}^2$$

$$4^2 + 2 \cdot 4 \cdot 2x + 2^2 = 16(1-3x)$$

$$16 + 16x + 4x^2 = 16 - 48x$$

$$4x^2 + 16x + 48x = 0$$

$$4x^2 + 64x = 0$$

$$4x(x+16) = 0$$

$$4x=0 \quad | \quad x+16=0$$

$$x=0 \quad | \quad x=-16$$

$$29) \sqrt{3x+10} + \sqrt{9x+7} = 9$$

13

$$\sqrt{3x+10} = 9 - \sqrt{9x+7}$$

SOBS

$$3x+10 = 81 - 2 \cdot 9 \cdot \sqrt{9x+7} + (\sqrt{9x+7})^2$$

$$3x+10 = 81 - 18\sqrt{9x+7} + 9x+7$$

$$3x+10 - 9x - 7 - 81 = -18\sqrt{9x+7}$$

$$-6x - 78 = -18\sqrt{9x+7}$$

SOBS

$$3x + (-6x - 78) = (-18\sqrt{9x+7})$$

$$-3(x+26) = -18\sqrt{9x+7}$$

SOBS

$$x^2 + 144 - 24x = 9(9x+7)$$

$$x^2 + 144 - 24x = 81x + 63$$

$$\Rightarrow x^2 - 24x - 81x + 144 - 63 = 0$$

$$\Rightarrow x^2 - 105x + 81 = 0$$

$$25/6/18 \quad 30) \sqrt{x^2-16} - (x-4) = \sqrt{x^2-5x+4}$$

Consider $x^2 - 5x + 4$, $x^2 - 16$
 $\Rightarrow x^2 - 4x - x + 4$ $x^2 - 4^2$
 $\Rightarrow x(x-4) - 1(x-4)$ $(x-4)(x+4)$
 $\Rightarrow (x-4)(x-1)$
 $\Rightarrow x = 4, 1$

$$\sqrt{(x-4)(x+4)} - \sqrt{(x-4)^2} = \sqrt{(x-4)(x-1)}$$

$$\sqrt{(x-4)} (\sqrt{x+4} - \sqrt{x-4}) = \sqrt{x-4} \sqrt{x-1}$$

SOBS

$$x+4 - x-4 = x-1$$

$$\Rightarrow x-4 = x-1$$

$$\Rightarrow x-4 = x-1$$

$$\sqrt{x+4} - 2\sqrt{(x+4)(x-4)} + \sqrt{x-4} = \sqrt{x-1}$$

$$x+4 - 2\sqrt{x^2-16} + x-4 = x-1$$

$$2x - 2 + 1 = 2\sqrt{x^2-16}$$

$$x+1 = 2\sqrt{x^2-16}$$

SOBS

$$(x+1)^2 = 4(x^2-16)$$

$$x^2 + 2x + 1 = 4(x^2 - 16)$$

$$x^2 + 2x + 1 = 4x^2 - 64$$

$$\Rightarrow x^2 + 2x + 1 - 4x^2 + 64 = 0$$

$$\Rightarrow x - 3x^2 + 2x + 65 = 0$$

$$\Rightarrow 3x^2 - 2x - 65 = 0$$

$$\Rightarrow 3x^2 - 15x + 13x - 65 = 0$$

$$\Rightarrow 3x(x-5) + 13(x-5) = 0$$

$$\Rightarrow (x-5)(3x+13) = 0$$

$$x=5 \quad | \quad x=-13/3$$

$$31) \sqrt{x^2-8x+15} + \sqrt{x^2+2x-15} = \sqrt{4x^2-18x+18}$$

$$\sqrt{(x-5)(x-3)} + \sqrt{(x+5)(x-3)} = \sqrt{2(2x-3)(x-3)}$$

$$\sqrt{(x-3)} (\sqrt{x-5} + \sqrt{x+5}) = \sqrt{x-3} (\sqrt{2(2x-3)})$$

SOBS

$$(\sqrt{x-5})^2 + 2(\sqrt{x-5})(\sqrt{x+5}) + (\sqrt{x+5})^2 = 2(2x-3)$$

$$x^2+25-10x + 2(\sqrt{x^2-25}) + x^2+25+10x = 4x-6$$

$$2x^2+50+2(\sqrt{x^2-25}) = 4x-6$$

$$x-5 + 2(\sqrt{x^2-25}) + x+5 = 4x-6$$

$$2x + 2\sqrt{x^2-25} = 2(2x-3)$$

$$x(x+\sqrt{x^2-25}) = x(2x-3)$$

$$x-2x+3 = -\sqrt{x^2-25}$$

$$-x+3 = -\sqrt{x^2-25}$$

SOBS

$$x^2-2(x)(3)+3^2 = \sqrt{x^2-25}^2$$

$$x^2-6x+9 = x^2-25$$

$$-6x = -25-9$$

$$-6x = -34$$

$$x = 34/6 = 17/3$$

$$32) \sqrt{x^2-3x+36} - \sqrt{x^2-3x+9} = 3$$

$$\Rightarrow \sqrt{x^2-3x+36} = 3 + \sqrt{x^2-3x+9}$$

SOBS

$$\sqrt{x^2-3x+36} = 3 + 2 \cdot 3 \cdot \sqrt{x^2-3x+9} + (\sqrt{x^2-3x+9})^2$$

$$x^2-3x+36 = 9 + 6\sqrt{x^2-3x+9} + x^2-3x+9$$

$$36-9-9 = 6\sqrt{x^2-3x+9}$$

$$18 = 6\sqrt{x^2-3x+9}$$

SOBS

$$3 = \sqrt{x^2-3x+9}$$

$$9 = x^2-3x+9$$

$$x^2-3x = 0$$

$$x(x-3) = 0$$

$$x = 0 \mid x = 3$$

$$33) \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2 \frac{1}{6}$$

SOBS

$$\left(\sqrt{\frac{x}{1-x}}\right)^2 + 2\sqrt{\frac{x}{1-x}}\sqrt{\frac{1-x}{x}} + \left(\sqrt{\frac{1-x}{x}}\right)^2 = \frac{13}{6^2}$$

$$\frac{x}{1-x} + \frac{1-x}{x} = \frac{169}{36}$$

$$\frac{x^2 + (1-x^2)}{(1-x)(x)} = \frac{169}{36}$$

$$36x^2 + 36(1-x^2) = (169-x^2)x$$

$$36x^2 + 36 - 36x^2 = 169x - 169x^2$$

$$36x^2 + 36(1+x^2-2x) = 169x - 169x^2$$

26/6/18

$$33) \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$$

S.O.B.S

$$\left(\sqrt{\frac{x}{1-x}}\right) + 2\sqrt{\frac{x}{1-x}}\sqrt{\frac{1-x}{x}} + \left(\sqrt{\frac{1-x}{x}}\right) = \left(\frac{13}{6}\right)^2$$

$$\frac{x}{1-x} + \frac{1-x}{x} = \frac{169}{36}$$

$$\frac{x^2 + (1-x^2)}{(1-x)(x)} = \frac{169}{36}$$

$$\Rightarrow \frac{x^2 + (1-2x+x^2)}{x-x^2} = \frac{169}{36}$$

$$\Rightarrow \frac{x^2 + 1 - 2x + x^2}{x-x^2} = \frac{169-2(36)}{36}$$

$$\Rightarrow \frac{2x^2 - 2x + 1}{x-x^2} = \frac{169-72}{36}$$

$$\Rightarrow (2x^2 - 2x + 1)36 = (x-x^2)97$$

$$\Rightarrow 72x^2 - 72x + 36 = 97x - 97x^2$$

$$\Rightarrow 72x^2 - 72x + 36 - 97x + 97x^2$$

$$\Rightarrow 169x^2 - 169x + 36 = 0$$

$$\Rightarrow 169x^2 - 52x - 117x + 36 = 0$$

$$\Rightarrow 13x(13x-4) - 9(13x-4) = 0$$

$$(13x-4) \cdot 9(13x-4) = 0$$

$$x = 4/13, 19/13$$

Formula method

$$169x^2 - 169x + 36 = 0$$

$$a = 169, b = -169, c = 36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \frac{-(-169) \pm \sqrt{169^2 - 4(169)(36)}}{2(169)}$$

$$\Rightarrow \frac{169 \pm \sqrt{169[169-4(36)]}}{2(169)}$$

$$\Rightarrow \frac{169 \pm \sqrt{169(25)}}{338} = \frac{169 \pm 65}{338}$$

$$x = \frac{169+65}{338}, x = \frac{169-65}{338}$$

$$= \frac{234}{338} = \frac{117}{169}$$

$$= 9/13$$

$$= \frac{104}{338} = \frac{52}{169}$$

$$= 4/13$$

$$34) \sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = 2\frac{1}{2}$$

$$\text{let } a = \sqrt{\frac{x}{x-3}}$$

$$\therefore \frac{1}{a} = \sqrt{\frac{x-3}{x}}$$

$$\Rightarrow a + \frac{1}{a} = \frac{5}{2}$$

$$\Rightarrow \frac{a^2 + 1}{a} = \frac{5}{2}$$

$$\Rightarrow 2a^2 + 2 = 5a$$

$$\Rightarrow 2a^2 - 5a + 2 = 0$$

$$\Rightarrow 2a^2 - 4a - a + 2 = 0$$

$$\Rightarrow 2a(a-2) - 1(a-2) = 0$$

$$(a-2)(2a-1) = 0$$

$$a = 2 \text{ or } \frac{1}{2}$$

Case 1:

$$\sqrt{\frac{x}{x-3}} = 2$$

$$\frac{x}{x-3} = 4$$

$$x = 4x - 12$$

$$-3x = -12$$

$$x = 4$$

Case 2:

$$\sqrt{\frac{x}{x-3}} = \frac{1}{2}$$

$$\frac{x}{x-3} = \frac{1}{4}$$

$$4x = x - 3$$

$$4x - x + 3 = 0$$

$$3x + 3 = 0$$

$$x + 1 = 0$$

$$x = -1$$

$$35) \sqrt{\frac{3x}{1+x}} + \sqrt{\frac{1+x}{3x}} = 2$$

$$\text{let } a = \sqrt{\frac{3x}{1+x}}$$

$$\frac{1}{a} = \sqrt{\frac{1+x}{3x}}$$

$$\Rightarrow a + \frac{1}{a} = 2$$

$$\Rightarrow a^2 + 1 = 2a$$

$$\Rightarrow a^2 - 2a + 1 = 0$$

$$\Rightarrow a^2 - a - a + 1 = 0$$

$$\Rightarrow a(a-1) - 1(a-1) = 0$$

$$\Rightarrow (a-1)(a-1) = 0$$

$$a = 1$$

Case 1:

$$\sqrt{\frac{3x}{1+x}} = 1$$

$$\frac{3x}{1+x} = 1$$

$$3x = 1+x$$

$$3x - x = 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Case 2:

$$\sqrt{\frac{3x}{1+x}} = 1$$

$$\frac{3x}{1+x} = 1$$

$$3x = 1+x$$

$$3x - x = 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

26/6/18

Completing Squares Method

* Step 1: Make the Coefficient of x^2 as 1

→ Take the constant to the other side

→ Add $(b/2)^2$ on both sides

→ If b is '-' apply $(a-b)^2$, if b is '+' apply $(a+b)^2$

i) $2x^2 - 3x - 2 = 0$

Divide by 2

$$\Rightarrow \frac{2x^2}{2} - \frac{3x}{2} - \frac{2}{2} = 0$$

$$\Rightarrow x^2 - 3/2x - 1 = 0$$

$$(b/2)^2 = (3/4)^2 = 9/16$$

$$x^2 - 3/2x + 9/16 = 1 + 9/16$$

$$x^2 - 3/2x + (3/4)^2 = \frac{16+9}{16}$$

$$(x - 3/4)^2 = 25/16$$

$$(x - 3/4)^2 = (5/4)^2$$

$$\Rightarrow x - 3/4 = \pm (5/4)$$

Case 1: $x - 3/4 = 5/4$, Case 2: $x - 3/4 = -5/4$

$$x - 3/4 = 5/4$$

$$x = 5/4 + 3/4$$

$$x = 8/4$$

$$x = 2$$

$$x = -5/4 + 3/4$$

$$x = -2/4$$

$$x = -1/2$$

ii) $5x^2 - 6x - 2 = 0$

Divide by 5

$$\frac{5x^2}{5} - \frac{6x}{5} - \frac{2}{5} = 0$$

$$\Rightarrow x^2 - 6/5x - 2/5 = 0$$

$$(b/2)^2 = \left(\frac{6/5}{2}\right)^2 = \left(\frac{6/10}{1}\right)^2 = \left(\frac{3}{5}\right)^2$$

$$x^2 - 6/5x + 9/25 = 2/5 + 9/25$$

$$x^2 - 6/5x + 9/25 = \frac{25+9}{25}$$

$$x^2 - 6/5x + 9/25 = 19/25$$

$$x^2 - 6/5x + \left(\frac{3}{5}\right)^2 = \left(\frac{\sqrt{19}}{5}\right)^2$$

$$(x - 3/5)^2 = \left(\frac{\sqrt{19}}{5}\right)^2$$

$$x - 3/5 = \pm \frac{\sqrt{19}}{5}$$

Case 1: $x - 3/5 = \frac{\sqrt{19}}{5}$, Case 2: $x - 3/5 = -\frac{\sqrt{19}}{5}$

$$\frac{5x-3}{5} = \frac{\sqrt{19}}{5}$$

$$5x-3 = \sqrt{19}$$

$$5x = \sqrt{19} + 3$$

$$5x = 5\left(\frac{\sqrt{19}+3}{5}\right)$$

$$x = \frac{\sqrt{19}+3}{5}$$

$$\frac{5x-3}{5} = -\frac{\sqrt{19}}{5}$$

$$5x-3 = -\sqrt{19}$$

$$5x = -\sqrt{19} + 3$$

$$5x = \frac{-\sqrt{19}+3}{5}$$

$$\text{iii) } 3x^2 + 5x - 2 = 0$$

Divide by 3

$$\frac{3x^2}{3} + \frac{5x}{3} - \frac{2}{3} = 0$$

$$\Rightarrow x^2 + \frac{5}{3}x - \frac{2}{3} = 0$$

$$(b/2)^2 = \left(\frac{5/3}{2}\right)^2 - \frac{-2/3}{1} = 0$$

$$(b/2)^2 = \left(\frac{5/3}{2}\right)^2 = (5/6)^2 = 25/36$$

$$\Rightarrow x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{2}{3} + \frac{25}{36}$$

$$\Rightarrow x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \frac{2}{3} + \frac{25}{36}$$

$$\Rightarrow x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \frac{24+25}{36}$$

$$\Rightarrow x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \left(\frac{7}{6}\right)^2$$

$$\Rightarrow (x + \frac{5}{6})^2 = \left(\frac{7}{6}\right)^2$$

$$\Rightarrow x + \frac{5}{6} = \pm \frac{7}{6}$$

Case 1: $x + \frac{5}{6} = \frac{7}{6}$

$$x = \frac{7}{6} - \frac{5}{6}$$

$$x = \frac{2}{6} = \frac{1}{3}$$

Case 2: $x + \frac{5}{6} = -\frac{7}{6}$

$$x = -\frac{7}{6} - \frac{5}{6}$$

$$x = -\frac{12}{6}$$

$$x = -2$$

$$\frac{27/6118}{4) x^2 - 3x - 10 = 0$$

$$(b/2)^2 = (3/2)^2 = 9/4$$

$$x^2 - 3x + 9/4 = 10 + 9/4$$

$$x^2 - 3x + 9/4 = \frac{40+9}{4}$$

$$x^2 - 3x + 9/4 = 49/4$$

$$x^2 - 3x + (3/2)^2 = (7/2)^2$$

$$(x - 3/2)^2 = (7/2)^2$$

$$x - 3/2 = \pm 7/2$$

Case 1:

$$x - 3/2 = 7/2$$

$$x = 7/2 + 3/2$$

$$x = 10/2$$

$$x = 5$$

Case 2:

$$x - 3/2 = -7/2$$

$$x = -7/2 + 3/2$$

$$x = -4/2$$

$$x = -2$$

Case 2

$$5) \sqrt{3}x^2 + 10x - 8\sqrt{3} = 0 \Rightarrow (4\sqrt{3}, 2/\sqrt{3})$$

$$6) 2x^2 + x - 1 = 0 \quad (1/2, -1)$$

$$7) x^2 - 6x + 9 = 0 \quad (-3, -3)$$

$$8) x^2 - 8x + 15 \quad (-5, -3)$$

$$9) \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

19

$$\frac{x-7 - (x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{x-7 - x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$-330 = 11[(x+4)(x-7)]$$

$$-330 = 11[x^2 - 3x - 28]$$

$$-30 = x^2 - 3x - 28$$

$$\Rightarrow x^2 - 3x - 28 + 30 = 0$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$(b/2)^y = (3/2)^y = 9/4$$

$$\Rightarrow x^2 - 3x + 9/4 = -2 + 9/4$$

$$\Rightarrow x^2 - 3x + (3/2)^y = \frac{-8+9}{4}$$

$$\Rightarrow x^2 - 3x + (3/2)^y = (1/4)^y$$

$$\Rightarrow (x - 3/2)^y = (1/2)^y$$

$$\Rightarrow x - 3/2 = \pm 1/2$$

Case 1:

$$\Rightarrow x - 3/2 = 1/2$$

$$\Rightarrow x = 1/2 + 3/2 = 4/2 = 2$$

Case 2:

$$x - 3/2 = -1/2$$

$$\Rightarrow x = -1/2 + 3/2$$

$$x = 2/2 = 1$$

$$8) x^2 - 8x + 15 = 0$$

$$(b/2)^y = (8/2)^y = 64/4$$

$$\Rightarrow x^2 - 8x + \frac{64}{4} = -15 + 64/4$$

$$\Rightarrow x^2 - 8x + \frac{64}{4} = \frac{-60 + 64}{4}$$

$$\Rightarrow x^2 - 8x + (8/2)^y = \frac{-24}{4}$$

$$(x - 8/2)^y = -1$$

$$x - 8/2 = \pm 1$$

Case 1: $x - 8/2 = 1$

$$x = 1 + 8/2$$

$$= 10/2$$

$$= 5$$

Case 2: $x - 8/2 = -1$

$$x = -1 + 8/2$$

$$= 6/2$$

$$= 3$$

* find the Quadratic Equation whose roots are 6, 7.

sol Given the two roots are $\alpha=6$, $\beta=7$.

WKT Quadratic Equation is $x^2 - (\text{sum})x + \text{product} = 0$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (6+7)x + 6(7) = 0$$

$$\Rightarrow x^2 - 13x + 42 = 0$$

2) find the Quadratic Equation whose roots are 11, -2

sol Given the two roots are $\alpha=11$, $\beta=-2$

WKT, Quadratic Equation is $x^2 - (\text{sum})x + \text{product} = 0$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (11-2)x + 11(-2) = 0$$

$$\Rightarrow x^2 - 9x - 22 = 0$$

3) find the Quadratic Equation whose roots are -5, -3

sol Given the two roots are $\alpha=-5$, $\beta=-3$

WKT, Quadratic Equation is $x^2 - (\text{sum})x + \text{product} = 0$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (-5-3)x + 5(-3) = 0$$

$$\Rightarrow x^2 - 8x + 15 = 0$$

4) find the Quadratic Equation whose roots are 5, 3

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (5+3)x + 5(3) = 0$$

$$\Rightarrow x^2 - 8x + 15 = 0$$

20 4) find the Quadratic Equation of $5+\sqrt{3}, 5-\sqrt{3}$

sol $\alpha + \beta = 5 + \sqrt{3} + 5 - \sqrt{3}$

$$= 10$$

$$\alpha\beta = (5+\sqrt{3})(5-\sqrt{3})$$

$$= 25 - 5\sqrt{3} + 5\sqrt{3} - 3$$

$$= 22$$

Quadratic Equation of the roots $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$x^2 - (10 + 22)x + 22 = 0$$

$$\Rightarrow x^2 - 32x + 22 = 0$$

$$\Rightarrow x^2 - 10x + 22 = 0$$

5) find α, β are the roots of Quadratic Equation $x^2 - 2x + 3 = 0$

21 then form an Equation whose roots are 1) $\alpha+3, \beta+3$.
2) $2\alpha+3\beta, 3\alpha+2\beta$

sol Given the Quadratic Equation is $x^2 - 2x + 3 = 0$

$$\therefore a=1, b=-2, c=3$$

$$\alpha + \beta = -b/a = 2/1 = 2$$

$$\alpha\beta = c/a = 3/1 = 3$$

① $\alpha+3, \beta+3$

$$\text{let } A = \alpha+3, B = \beta+3$$

$$\therefore \text{sum} = A+B = \alpha+3+\beta+3 = \alpha+\beta+6 = 2+6 = 8$$

$$\begin{aligned} AB &= (\alpha+3)(\beta+3) = \alpha\beta + 3\alpha + 3\beta + 9 = 3 + 3(\alpha+\beta) + 9 \\ &= 3 + 6 + 9 = 18 \end{aligned}$$

$\therefore$ The Quadratic Equation $x^2 - \text{sum}(x) + \text{Product} = 0$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (a+b)x + ab = 0$$

$$\Rightarrow x^2 - 8x + 18 = 0$$

2) $2\alpha + 3\beta, 3\alpha + 2\beta$

Sol let $C = 2\alpha + 3\beta, D = 3\alpha + 2\beta$

~~Sum $(\alpha + \beta)$~~

$$\text{Sum}(C+D) = 2\alpha + 3\beta + 3\alpha + 2\beta = 5\alpha + 5\beta$$

$$= 5(\alpha + \beta)$$

$$= 5(2) = 10$$

$$\text{product}(CD) = (2\alpha + 3\beta)(3\alpha + 2\beta) = 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2$$

$$= 13\alpha\beta + 6(\alpha^2 + \beta^2)$$

$$= 13\left(\frac{\alpha\beta}{1}\right) + 6(\alpha + \beta)^2 - 12\alpha\beta$$

$$= -3\alpha + 12 = 13\alpha\beta + 6(\alpha + \beta)^2 - 12\alpha\beta$$

$$= -3\alpha + 6(2)^2 = 6(2)^2 + \alpha\beta$$

$$= 24 + 3 = 27$$

$\therefore$ The Quadratic Equation $x^2 - \text{Sum}(x) + \text{product} = 0$

$$\Rightarrow x^2 - (\alpha + \beta)x + cd = 0$$

$$\Rightarrow x^2 - 10x + 27 = 0$$

* If α & β are the roots of the equation $x^2 - 5x + 6$ then form a Quadratic Equation whose roots are 1) $\alpha + 2, \beta + 2$ 2) $2\alpha - 3\beta, 3\alpha - 2\beta$

3) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Sol Given the Quadratic Equation is $x^2 - 5x + 6 = 0$

$$\therefore a = 1, b = -5, c = 6$$

$$\alpha + \beta = -b/a = 5/1 = 5$$

$$\alpha\beta = c/a = 6/1 = 6$$

1) $\alpha + 2, \beta + 2$

let $A = \alpha + 2, B = \beta + 2$

$$\therefore \text{sum} = A + B = \alpha + 2 + \beta + 2 = (\alpha + \beta) + 4 = 5 + 4 = 9$$

$$AB = (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$$

$$= 6 + 2(5) + 4 = 6 + 10 + 4 = 20$$

∴ The Quadratic Equation $x^2 - \text{sum}(x) + \text{product} = 0$

$$\Rightarrow x^2 - 9x + 20 = 0.$$

2) $2\alpha - 3\beta, 3\alpha - 2\beta$.

let $C = 2\alpha + 3\beta, D = 3\alpha + 2\beta$

$$\text{Sum}(C+D) = 2\alpha + 3\beta + 3\alpha + 2\beta$$

$$= 2\alpha + 2\beta + 3\alpha + 3\beta$$

$$= 2(\alpha + \beta) + 3(\alpha + \beta)$$

$$= 2(5) + 3(5) = 10 + 15 = 25$$

$$\begin{aligned} \text{Product} &: (2\alpha + 3\beta)(3\alpha + 2\beta) \\ &= 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2 \\ &= 6(\alpha^2 + \beta^2) + 13\alpha\beta \\ &= 6[(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta \\ &= 6[(5)^2 - 2(6)] + 13(6) \\ &= 6[25 - 12] + 78 = 6(13) + 78 \\ &= 78 + 78 = 156 \end{aligned}$$

3) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

let $E = \alpha/\beta, F = \beta/\alpha$

$$\text{Sum} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(5)^2 - 2(6)}{6} = \frac{25 - 12}{6}$$

$$\text{Product} = \frac{\alpha\beta}{\beta\alpha} = \frac{6}{6} = 1$$

$$\text{Equation} : x^2 - \frac{13}{6}x + 1 = 0$$

* If α, β are root of the Equation $2x^2 - 4x + 1 = 0$ Find the

Equation whose root are $\alpha^2 + \beta, \alpha + \beta^2$

Sol Quadratic Equation is $2x^2 - 4x + 1 = 0$

$$\text{Sum}(\alpha + \beta) = -b/a = -\frac{(-4)}{2} = 2$$

$$\text{Product}(\alpha\beta) = c/a = 1/2$$

let $A = \alpha^2 + \beta, B = \alpha + \beta^2$

$$A+B = \alpha^2 + \beta + \alpha + \beta^2$$

$$= \alpha^2 + \beta^2 + (\alpha + \beta)$$

$$\begin{aligned}
 &= (\alpha + \beta)^{\gamma} - 2\alpha\beta + (\alpha + \beta) \\
 &= (2)^{\gamma} - 2(1/2) + (2) \\
 &= 4 - 1 + 2 \\
 &= 5.
 \end{aligned}$$

Product $AB = (\alpha^{\gamma} + \beta)(\alpha + \beta)^{\gamma} = \alpha^3 + \alpha^{\gamma}\beta^{\gamma} + \alpha\beta + \beta^3$

$$= \alpha^3 + \beta^3 + (\alpha\beta)^{\gamma} + \alpha\beta$$

$$= (\alpha + \beta)(\alpha^{\gamma} - \alpha\beta + \beta^{\gamma}) + (\alpha\beta)^{\gamma} + \alpha\beta$$

$$= (\alpha + \beta)((\alpha + \beta)^{\gamma} - 2\alpha\beta - \alpha\beta) + (\alpha\beta)^{\gamma} + \alpha\beta$$

$$= (\alpha^3 + \beta^3) + (2\alpha\beta)^{\gamma} + \alpha\beta$$

$$= (\alpha + \beta)[(\alpha + \beta)^{\gamma} - 3\alpha\beta] + (\alpha\beta)^{\gamma} + \alpha\beta$$

$$= 2[2^{\gamma} - 3(1/2)] + (1/2)^{\gamma} + 1/2$$

$$= 2[4 - 3/2] + 1/4 + 1/2$$

$$= 2\left[\frac{8-3}{2}\right] + 1/4 + 1/2$$

$$= 5 + 1/4 + 1/2$$

$$= \frac{20+1+2}{4} = 23/4.$$

Equation: $x^{\gamma} - 5x + 23/4 = 0 \Rightarrow 4x^{\gamma} - 20x + 23 = 0$

* Equation $x^{\gamma} - 5x + 2$, a) $\alpha+2, \beta+2$, b) $3\alpha+5\beta, 5\alpha+3\beta$, c) $\alpha/\beta, \beta/\alpha$, d) $\alpha^{\gamma}+\beta, \alpha+\beta^{\gamma}$

Sum $(\alpha + \beta) = -b/a = 5/1$

Double

$$(\alpha\beta) = c/a = 2.$$

let $A = \alpha + 2, B = \beta + 2$

a) $A + B = \alpha + 2 + \beta + 2 = \alpha + \beta + 4 = 5 + 4 = 9$

$AB = (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4 = \alpha\beta + 2(\alpha + \beta) + 4 = 0 \Rightarrow 2 + 2(5) + 4 = 16$
 $2 + 10 + 4 = 16 \Rightarrow 16$

b) $3\alpha + 5\beta + 5\alpha + 3\beta = 3(\alpha + \beta) + 5(\alpha + \beta)$

$$= 3(5) + 5(5) = 15 + 25 = 40$$

$$c) \alpha/\beta + \beta/\alpha = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$\text{Sum} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(5)^2 - 2(2)}{2}$$

$$= \frac{25 - 4}{2} = 21/2$$

$$\text{product} = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha}$$

$$= \frac{\alpha\beta}{\beta\alpha} = \frac{2}{2} = 1$$

$$\therefore \text{Equation } x^2 + \frac{21}{2}x + 1 = 0$$

$$\Rightarrow 2x^2 + 21x + 2 = 0$$

$$d) \alpha^2 + \beta, \alpha + \beta^2$$

$$\text{Sum} = \alpha^2 + \beta + \alpha + \beta^2$$

$$= (\alpha^2 + \beta^2) + (\alpha + \beta)$$

$$= (\alpha + \beta)^2 - 2\alpha\beta + (\alpha + \beta)$$

$$= (5)^2 - 2(2) + (5)$$

$$= 25 - 4 + 5 = 26$$

$$\text{product} = (\alpha^2 + \beta)(\alpha + \beta^2)$$

$$= \alpha^3 + \alpha^2\beta^2 + \beta\alpha + \beta^3$$

$$= (\alpha^3 + \beta^3) + (\alpha^2\beta^2) + \alpha\beta$$

$$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] + (\alpha\beta)^2 + \alpha\beta$$

$$= 5[(5)^2 - 3(2)] + (2)^2 + 2$$

$$= 5[25 - 6] + 4 + 2$$

$$= 105 + 6 = 111$$

$$\text{Equation: } x^2 + 26x + 111 = 0$$

$$4) \frac{41x}{x^2 - 6x + 9} = 0$$

$$(b/2)^y = (6/2)^y = 9$$

$$x^y - 6x + 9 = -9 + 9$$

$$x^y - 6x + 9 = 0$$

$$x^y - 3x - 3x + 9 = 0$$

$$x(x-3) - 3(x-3) = 0$$

$$(x-3)(x-3) = 0$$

$$x = 3, 3$$

$$6) 2x^y + x - 1 = 0$$

Divide by '2' on b.s

$$\frac{2x^y}{2} + \frac{x}{2} - \frac{1}{2} = 0$$

$$x^y + x/2 - 1/2 = 0$$

$$(b/2)^y = (1/2)^y = 1/16$$

$$x^y + x/2 + 1/16 = 1/2 + 1/16$$

$$\frac{16x^y + 8x + 1}{16} = \frac{8x + 1}{16}$$

$$16x^y + 8x + 1 = 8x + 1$$

$$16x^y + 8x -$$

$$x^y + x/2 + (1/4)^y = \frac{9}{16}$$

$$(x + 1/4)^y = (3/4)^y$$

$$x + 1/4 = \pm 3/4$$

Case 1: $x + 1/4 = 3/4$

$$x = 2/4$$

$$x = 1/2$$

Case 2: $x + 1/4 = -3/4$

$$x = -3/4 - 1/4$$

$$x = -1$$

$$x = -1$$

$$5) \sqrt{3}x^y + 10x - 8\sqrt{3} = 0$$

$$\frac{10}{2\sqrt{3}}$$

Divide by $\sqrt{3}$

Doubt

$$\frac{\sqrt{3}x^y}{\sqrt{3}} + \frac{10}{\sqrt{3}}x - \frac{8\sqrt{3}}{\sqrt{3}} = 0$$

$$x^y + \frac{10}{\sqrt{3}}x - 8 = 0$$

$$(b/2)^y = \left(\frac{10/\sqrt{3}}{2}\right)^y = \left(\frac{10}{2\sqrt{3}}\right)^y$$

$$x^y + \frac{10}{\sqrt{3}}x + \left(\frac{10}{2\sqrt{3}}\right)^y = 8 + \left(\frac{10}{2\sqrt{3}}\right)^y$$

$$x^y + \frac{10}{\sqrt{3}}x + \left(\frac{10}{2\sqrt{3}}\right)^y = \frac{96 + 100}{12}$$

$$\left(x^y + \frac{10}{2\sqrt{3}}\right)^y = \frac{196}{12}$$

$$\left(x + \frac{10}{2\sqrt{3}}\right)^y = \left(\frac{\sqrt{196}}{2\sqrt{3}}\right)^y$$

$$x + \frac{10}{2\sqrt{3}} = \pm \frac{\sqrt{196}}{2\sqrt{3}} \left(\frac{49 \times 1}{4 \times 3}\right)^{1/y}$$

Case 1: $x + \frac{10}{2\sqrt{3}} = \frac{\sqrt{196}}{2\sqrt{3}} \frac{14}{2\sqrt{3}}$

Case 2:

$$x = \frac{\sqrt{196}}{2\sqrt{3}} + \frac{14}{2\sqrt{3}}$$

$$x + \frac{10}{2\sqrt{3}} = -\frac{14}{2\sqrt{3}}$$

$$x + \frac{10}{2\sqrt{3}} = \frac{14}{2\sqrt{3}}$$

$$x = -\frac{14}{2\sqrt{3}} - \frac{10}{2\sqrt{3}}$$

$$x = \frac{14}{2\sqrt{3}} - \frac{10}{2\sqrt{3}}$$

$$x = -\frac{24}{2\sqrt{3}}$$

$$x = 4/2\sqrt{3}$$

$$x = -\frac{12}{\sqrt{3}}$$

28/6/18

II. PROGRESSIONS

* An A.P is the series whose terms either increase or decrease by a fixed number called as common difference (d) Eg: 1, 3, 5, 7, 9, ...

Eg: -11, -9, -7, -5, ..., 1, -1, -3, -5, ...

Therefore $Ap = a, a+d, a+2d, \dots + a+(n-1)d$.

$$t_1 = a = 1^{\text{st}} \text{ term}$$

$$t_2 = a+d$$

$$t_3 = a+2d$$

$$t_n = a+(n-1)d$$

d = common difference

$$d = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{n}{2} [t_1 + t_n]$$

3 terms $a-d, a, a+d$

4 terms $a-3d, a-2d, a+d, a+3d$

5 terms $a-2d, a-d, a+d, a+2d$

* Sum of 1^{st} n natural numbers

$$S_n = 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$S_n^2 = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_n^3 = 1^3+2^3+3^3+\dots+n^3 = \frac{n^2(n+1)^2}{4}$$

* If a, b, c are in A.P

sol $a, b, c \rightarrow A.P$

$$t_1 \quad t_2 \quad t_3$$

$$t_2 - t_1 = t_3 - t_2$$

$$b - a = c - b$$

$$2b = c + a$$

$$b = \frac{c+a}{2}$$

$\therefore b$ is called as the arithmetic mean of a & c . hence the arithmetic mean of any two numbers is $\frac{x+y}{2}$

* 1) find the 20th term and sum to the 20 terms of the series $3, 5, 7, \dots$

$$t_1 = a = 3$$

$$d = t_2 - t_1 = 5 - 3 = 2$$

We know that, $t_n = a + (n-1)d$

$$\begin{aligned} t_{20} &= 3 + (20-1)2 \\ &= 3 + 38 = 41 \end{aligned}$$

We know that, $S_n = \frac{n}{2} [2a + (n-1)d]$ (or) $\frac{n}{2} [t_1 + t_n]$

$$S_{20} = \frac{20}{2} [2(3) + (20-1)2] \quad \frac{20}{2} [t_1 + t_{20}]$$

$$= 10 [6 + 38]$$

$$10 [3 + 41]$$

$$= 440$$

$$= 440$$

2) Find the 35th term & the sum to the 40 terms of 7, 11, 15, ...

Sol. 7, 11, 15, ...

$$t_1 = a = 7$$

$$d = t_2 - t_1 = 11 - 7 = 4$$

$$\begin{array}{r} 34 \\ 14 \\ \hline 136 \end{array}$$

WKT, $t_n = (a + (n-1)d)$

$$t_{35} = 7 + (35-1)4$$

$$= 7 + (34)4$$

$$t_{35} = 7 + 136 = 143$$

WKT, $S_n = \frac{n}{2} [2a + (n-1)d]$

$$S_{40} = \frac{40}{2} [2(7) + (40-1)4]$$

$$= 20 [14 + 39(4)]$$

$$= 20 [170] = 3400$$

Sol.
3) Find the sum to 17 terms of the A.P. $5\frac{1}{2}, 6\frac{3}{4}, 8, \dots$

WKT $a = 5\frac{1}{2} = 11/2$

$$d = t_2 - t_1 = 6\frac{3}{4} - 5\frac{1}{2} = \frac{27}{4} - \frac{11}{2} = \frac{27-22}{4} = \frac{5}{4}$$

WKT $S_n = \frac{n}{2} [2a + (n-1)d]$

$$S_{17} = \frac{17}{2} [2(11/2) + (17-1)5/4]$$

$$= 8.5 \left[11 + 10 \left(\frac{5}{4} \right) \right]$$

$$= 8.5 \left[\frac{22+40}{2} \right] = 8.5 \left[\frac{62}{2} \right] = 263.5$$

4) find the n^{th} term of the series 4, 9, 14, ... Hence find t_{19}

sol

$$a = 4$$

$$d = t_2 - t_1 = 9 - 4 = 5$$

$$\begin{aligned} \text{WKT } t_n &= a + (n-1)d \\ &= 4 + (n-1)5 \\ &= 4 + 5n - 5 \\ &= 5n - 1 \end{aligned}$$

$$\begin{aligned} \therefore t_{19} &= 5(19) - 1 \\ &= 95 - 1 = 94 \end{aligned}$$

5) find the r^{th} term 2, 8, 14, ... also find 15th term & sum to 15 terms

sol

$$a = 2$$

$$d = t_2 - t_1 = 8 - 2 = 6$$

$$\begin{aligned} t_r &= a + (r-1)d \\ &= 2 + (r-1)6 \\ &= 2 + 6r - 6 \\ &= 6r - 4 \end{aligned}$$

$$\therefore t_{15} = 6(15) - 4 = 90 - 4 = 86$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{15} = \frac{15}{2} [2(2) + (15-1)6]$$

$$= \frac{15}{2} [4 + (14)6]$$

$$= \frac{15}{2} [4 + 84] = \frac{1320}{2} = 660$$

6) $3\frac{1}{2}, 7, 10\frac{1}{2} \dots$ find Sum to 17 terms

3

$$a = 3\frac{1}{2}$$

$$d = 7 - 3\frac{1}{2} = 3\frac{1}{2}$$

$$S_n = \left[2a + (n-1)d \right] \frac{n}{2}$$

$$S_{17} = \frac{17}{2} \left[2\left(3\frac{1}{2}\right) + (17-1)7\frac{1}{2} \right]$$

$$= \frac{17}{2} \left[\frac{14}{2} + \frac{112}{2} \right]$$

$$= \frac{17}{2} [63] = \frac{1071}{2} = 535.5$$

7) find the n^{th} term of $2, 5, 8 \dots$ Also find 23^{rd} term & Sum to 23 terms

sol

$$a = 2$$

$$d = t_2 - t_1 = 5 - 2 = 3$$

$$\begin{aligned} t_{23} = t_n &= a + (n-1)d \\ &= 2 + (n-1)3 \\ &= 2 + 3n - 3 \\ &= 3n - 1 \end{aligned}$$

$$t_{23} = 3(23) - 1 = 68$$

$$\begin{aligned} S_{23} &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{23}{2} [2(2) + (23-1)3] \\ &= \frac{23}{2} [4 + 66] = 805 \end{aligned}$$

8) If the 4^{th} term of Ap is 21 & 14^{th} term is 81 find 25^{th} term

$$\text{Given } t_4 = 21, t_{14} = 81$$

$$t_4 = a + 3d = 21$$

$$a = 21 - 3d \quad \text{--- (1)}$$

$$t_{14} = a + 13d = 81$$

$$\Rightarrow 21 - 3d + 13d = 81$$

$$\Rightarrow 21 + 10d = 81$$

$$10d = 81 - 21$$

$$10d = 60$$

$$d = 6$$

∴ Sub $d=6$ in Equ ①

$$\rightarrow a = 21 - 3d$$
$$= 21 - 3(6)$$

$$= 21 - 18$$

$$\therefore a = 3$$

$$t_{25} = a + 24d =$$

$$t_{25} = a + (n-1)d$$
$$= a + (n-1)6$$

$$= 3 + 6n - 6$$

$$= 6n - 3$$

$$t_{25} = 6(25) - 3 = 147$$

21/18

✓ 9) The 3rd term is 18 & the 7th term is 30 find t_{28} ?

$$\text{Given } t_3 = 18, t_7 = 30$$

$$t_3 = a + 2d = 18$$

$$a = 18 - 2d \quad \text{--- ①}$$

$$t_7 = a + 6d = 30$$

$$a = 30 - 6d \quad \text{--- ②}$$

$$18 - 2d = 30 - 6d$$

$$4d = 12$$

$$d = 3$$

∴ Sub $d=3$ in Equ ①

$$a = 18 - 6$$

$$a = 12$$

$$t_{28} = a + (n-1)d$$

$$= 12 + (n-1)3$$

$$= 12 + 3n - 3 = 3n + 9$$

$$\begin{aligned}
 t_{28} &= 3n + 9 \\
 &= 3(28) + 9 \\
 &= 93
 \end{aligned}$$

✓
10) The 14th term is 6 and 6th term is 14 find t_{95}

Given $t_{14} = 6$, $t_6 = 14$

$$\begin{aligned}
 t_{14} &= a + 13d = 6 & t_6 &= 14 \Rightarrow t_6 = a + 5d = 14 \\
 a &= 136 - 13d \text{ --- ①} & \Rightarrow a &= 14 - 5d \\
 & & \Rightarrow 6 - 13d &= 14 - 5d \\
 & & \Rightarrow -8d &= 8 \\
 & & d &= -1
 \end{aligned}$$

∴ substitute $d = -1$ in ①

$$\begin{aligned}
 a &= 6 + 13 \\
 a &= 19
 \end{aligned}$$

$$\begin{aligned}
 t_n &= a + (n-1)d \\
 &= 19 + (-n+1) \\
 &= 20 - n
 \end{aligned}
 \quad \left| \quad \begin{aligned}
 t_{95} &= 20 - 95 \\
 &= -75
 \end{aligned}
 \right.$$

11) If the 4th term is 0 & 54th term is -125 find t_{42} .

Given $t_4 = 0$, $t_{54} = -125$

$$\begin{aligned}
 t_4 &= a + 3d = 0 & t_{54} &= a + 53d = -125 \\
 a &= -3d \text{ --- ①} & \Rightarrow a &= -125 - 53d
 \end{aligned}$$

$$\Rightarrow -3d = -125 - 53d$$

$$\begin{aligned}
 \Rightarrow +50d &= -125 \\
 d &= \frac{-125}{50} = -\frac{5}{2}
 \end{aligned}$$

∴ substitute $d = -5/2$ in ①

$$\begin{aligned}
 a &= -3/2(-5) \\
 &= 15/2
 \end{aligned}$$

$$\begin{aligned}
 t_n &= a + (n-1)d \\
 &= \frac{15}{2} + (n-1)\left(-\frac{5}{2}\right) = \frac{15}{2} + \frac{-5n+5}{2} = \frac{20-5n}{2}
 \end{aligned}$$

$$\begin{aligned}
 t_{42} &= \frac{20-42(5)}{2} \\
 &= \frac{20-210}{2} \\
 &= \frac{-190}{2} = -95
 \end{aligned}$$

$$\begin{array}{r}
 42 \\
 2 \overline{) 84} \\
 \underline{84} \\
 0
 \end{array}$$

$$12) t_2 = 6, t_5 = 15, t_{20} = ?$$

$$\text{Given } t_2 = 6, t_5 = 15$$

$$t_2 = a + d = 6 \quad t_5 = a + 4d = 15$$

$$\Rightarrow a = 6 - d \quad \Rightarrow a = 15 - 4d$$

— ①

$$6 - d = 15 - 4d$$

$$a = 6 - 3$$

$$3d = 9$$

$$a = 3$$

$$d = 3$$

$$t_n = a + (n-1)d$$

$$t_{20} = 3(20)$$

$$= 3 + (n-1)3$$

$$= 60$$

$$= 3 + 3n - 3$$

$$= 3n$$

13) Which term of the series 4, 9, 14 is 94

7

$$\text{Given } t_1 = 4$$

$$d = t_2 - t_1 = 9 - 4 = 5$$

$$\text{W K T } t_n = a + (n-1)d$$

$$\text{Given } t_n = 94$$

$$94 = 4 + (n-1)5$$

$$= 5n - 1$$

$$95 = 5n$$

$$n = 19$$

14) Which term of the series 9, 11, 13 is 99

$$\text{Given } t_1 = 9$$

$$d = t_2 - t_1 = 11 - 9 = 2$$

$$\text{W K T } t_n = a + (n-1)d$$

$$t_n = 99$$

$$99 = 9 + 2n - 2$$

$$99 = 7 + 2n$$

$$92 = 2n \Rightarrow n = 46$$

15) Which term of the series 2, 4, 6 ... is 92 & also find the sum of those terms

$$t_1 = 2$$

$$d = 4 - 2 = 2$$

WKT

$$t_n = a + (n-1)d$$

$$t_n = 92$$

$$92 = 2 + 2n - 2$$

$$2n = 92 \Rightarrow n = 46$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{46}{2} [2(2) + 2n - 2]$$

$$= 23 [2n + 2]$$

$$= 23 [2(46) + 2] = 23(92 + 2)$$

$$= 23(94)$$

$$= 2162$$

16) If the last term of the series $2x, 5x/3, 4x/3 \dots$ is 0 find the

no. of terms

$$t_1 = 2x, t_2 = \frac{5x}{3}, d = t_2 - t_1 = \frac{5x}{3} - 2x = \frac{5x - 6x}{3} = -\frac{x}{3}$$

$$d = -\frac{x}{3}$$

$$t_n = 0$$

$$t_n = a + (n-1)d$$

$$= 2x + (n-1) \cdot -\frac{x}{3}$$

$$= \frac{2x - nx + x}{3} = \frac{6x - nx + x}{3} \Rightarrow 5x = nx - x$$

$$n = 7$$

✓ 17) find 'n' $10 + 20 + 30 + \dots + n = 550$

9 $S_n = 550, t_1 = 10, d = 20 - 10 = 10$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$550 = \frac{n}{2} [2(10) + (n-1)10]$$

$$= \frac{n}{2} [20 + 10n - 10]$$

$$= \frac{n}{2} [10n + 10]$$

$$1100 = 10n^2 + 10n$$

$$\Rightarrow 10n^2 + 10n - 1100 = 0$$

$$\frac{-10 \pm \sqrt{100 - 4(1100)(10)}}{2(10)} =$$

$$10(n^2 + n - 110) = 0$$

$$n^2 + n - 110 = 0$$

$$n = \frac{-1 \pm \sqrt{1 - 440}}{2}$$

$$= \frac{-1 \pm \sqrt{339i}}{2}$$

18) $12 + 16 + 20 + \dots + n = 208$ find 'n'

$$S_n = 208, t_1 = 12, d = 16 - 12 = 4$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$208 = \frac{n}{2} [2(12) + (n-1)4]$$

$$= \frac{n}{2} [24 + 4n - 4]$$

$$416 = 20n + 4n^2$$

$$4n^2 + 20n - 416 = 0$$

$$\frac{-20 \pm \sqrt{400 - 4(-416)4}}{2(4)}$$

$$= \frac{-20 \pm 84}{8}, \frac{-20 - 84}{8} = 8, -13$$

$$\frac{-20 \pm \sqrt{400 + 6656}}{8} = \frac{-20 \pm \sqrt{7056}}{8} = \frac{-20 \pm 84}{8}$$

19) $2 + 5 + 8 + \dots + n = 950$ find 'n'

10 $S_n = 950, a = 2, d = 5 - 2 = 3$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$950 = \frac{n}{2} [2(2) + (n-1)3]$$

$$= \frac{n}{2} [4 + 3n - 3]$$

$$950 = \frac{n}{2} [3n + 1]$$

$$1900 = 3n^2 + n$$

$$\Rightarrow 3n^2 + n - 1900 = 0$$

$$\Rightarrow 3n^2 + 76n - 75n - 1900 = 0$$

$$\Rightarrow 3n(n + 76) - 75n - 1900 = 0$$

$$\Rightarrow 3n^2 - 75n + 76n - 1900 = 0$$

$$\Rightarrow 3n(n - 25) + 76(n - 25) = 0$$

$$\Rightarrow (n - 25)(3n + 76) = 0$$

$$\Rightarrow n = 25, n = -76/3$$

$$\begin{array}{r} 5 \overline{) 5700} \\ 5 \overline{) 1140} \\ 57 \overline{) 228} \\ 3 \overline{) 97} \\ 19 \overline{) 19} \end{array}$$

20) $5 + \frac{23}{3} + \frac{31}{3} + \dots + n = 400$ find 'n'

"

$$S_n = 400, a = 5, d = \frac{23}{3} - 5 = \frac{23 - 15}{3} = \frac{8}{3}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$400 = \frac{n}{2} [2(5) + (n-1)\frac{8}{3}]$$

$$400 = \frac{n}{2} [10 + \frac{8}{3}n - \frac{8}{3}]$$

$$400 = \frac{n}{2} [\frac{30 + 8n - 8}{3}]$$

$$400 = \frac{n}{2} [\frac{22 + 8n}{3}]$$

$$400 = \frac{n}{2} \left[\frac{8n+22}{3} \right]$$

$$2400 = 8n^2 + 22n$$

$$\Rightarrow 8n^2 + 22n - 2400 = 0$$

$$\Rightarrow \frac{-22 \pm \sqrt{(22)^2 - 4(8)(-2400)}}{2(8)}$$

$$\Rightarrow \frac{-22 \pm \sqrt{+77284}}{16}$$

$$\Rightarrow \frac{-22 \pm 278}{16}$$

Case 1)

$$n = \frac{-22 + 278}{16}$$

$$= 16$$

Case 2 :

$$n = \frac{-22 - 278}{16}$$

$$= -18.75$$

Q1) $5 + 10 + 15 + \dots + n = 455$ find 'n'

$$S_n = 455, a = 5, d = 10 - 5 = 5$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$455 = \frac{n}{2} [2(5) + (n-1)5]$$

$$455 = \frac{n}{2} [10 + 5n - 5]$$

$$455 = \frac{n}{2} [5n + 5]$$

$$910 = 5n^2 + 5n$$

$$\Rightarrow 5n^2 + 5n - 910 = 0$$

$$\Rightarrow n^2 + n - 182 = 0$$

$$n = \frac{-1 \pm \sqrt{1 - 4(1)(-182)}}{2(1)}$$

$$n = \frac{-1 \pm 27}{2}$$

$$= \frac{-1 + 27}{2} = \frac{26}{2} = 13$$

$$n = \frac{-1 - 27}{2}$$

$$= -14$$

$$\begin{array}{r} 8 \overline{) 19200} \\ 5 \overline{) 3850} \\ 5 \overline{) 770} \\ 2 \overline{) 134} \\ 67 \overline{) 67} \\ 1 \end{array}$$

$$S_n = 3n^2 + 4n$$

$$3 + 4 = 7$$

$$12 + 8 = 20$$

$$7, 13, 19$$

22) $t_3 = 18, t_7 = 30$ find t_{17} & S_{17}

12 $t_3 = a + 2d = 18, t_7 = a + 6d = 30$

$$a = 18 - 2d$$

$$a = 30 - 6d$$

$$a = 18 - 6$$

$$18 - 2d = 30 - 6d$$

$$= 12$$

$$4d = 12$$

$$t_n = a + (n-1)d$$

$$d = 3$$

$$= 12 + (n-1)3$$

$$t_{17} = 9 + 3(17)$$

$$= 12 + 3n - 3$$

$$= 9 + 51$$

$$= 9 + 30$$

$$= 60$$

$$S_{17} = \frac{17}{2} [2(12) + (17-1)3]$$

$$= \frac{17}{2} [24 + 48]$$

$$= \frac{17}{2} [72] = 17(36) = 612$$

23) $a = 8, d = 2, S_n = 152, t_n = ? + 13$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$152 = \frac{n}{2} [2(8) + (n-1)2]$$

$$= \frac{n}{2} [-16 + 2n - 2]$$

$$= \frac{n}{2} [-18 + 2n]$$

$$152 = \frac{n}{2} [2n - 18]$$

$$104 = 2n^2 - 18n$$

$$2n^2 - 18n + 104 = 0$$

$$\frac{-(-18) \pm \sqrt{(-18)^2 - 4(2)(104)}}{2(2)} = \frac{18 \pm \sqrt{324 - 832}}{4} = \frac{18 \pm 34}{4}$$

$$n = \frac{18+34}{4}$$

$$= \frac{52}{4}$$

$$= 13$$

$$n = \frac{18-34}{4}$$

$$= \frac{-16}{4}$$

$$= -4$$

$$t_n = a + (n-1)d$$

$$= -8 + (13-1)2$$

$$= -8 + (12)2$$

$$= 24 - 8 = 16$$

24) $t_n = 5n - 1$, $S_{17} = ?$

13 $t_1 = 5(1) - 1 = 5 - 1 = 4$ $\therefore a = 4$

$$t_2 = 5(2) - 1 = 10 - 1 = 9$$

$$\therefore d = t_2 - t_1 = 9 - 4 = 5$$

$$\therefore d = 5$$

$$S_{17} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{17}{2} [2(4) + (17-1)5]$$

$$= \frac{17}{2} [188 + 80 - 5] = \frac{17}{2} [3 + 80] = 17(44) = 748$$

✓ 25) $t_p = 5p + 2$, $S_{23} = ?$

$$t_1 = 5(1) + 2 = 7$$

$$t_2 = 5(2) + 2 = 12$$

$$\therefore d = t_2 - t_1 = 12 - 7 = 5$$

$$S_{23} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{23}{2} [2(7) + (23-1)5]$$

$$= \frac{23}{2} [14 + (22)5]$$

$$= \frac{23}{2} [14 + (22)5] = 1426$$

$$26) t_r = 5r + 2, S_{12} = ?$$

$$t_r = 5(1) + 2 = 7$$

$$t_2 = 5(2) + 2 = 10 + 2 = 12$$

$$\therefore d = t_2 - t_1 = 12 - 7 = 5$$

$$S_{12} = \frac{n}{2} [2a + (n-1)d] = \frac{12}{2} [2(7) + (12-1)5] \\ = 6[14 + (11)5] = 6[69] = 414$$

$$27) t_r = 3r + 5, S_{25} = ?$$

$$t_r = 3r + 5$$

$$t_1 = 3(1) + 5 = 3 + 5 = 8$$

$$t_2 = 3(2) + 5 = 6 + 5 = 11$$

$$d = 11 - 8 = 3$$

$$S_{25} = \frac{n}{2} [2a + (n-1)d] \\ = \frac{25}{2} [2(8) + 24(3)] \\ = \frac{25}{2} [16 + 72] = \frac{25}{2} [88] = 25(44) = 1100$$

$$\checkmark 28) t_p = p/7 + 2, S_{35} = ?$$

$$14 \quad t_p = p/7 + 2 = 1/7 + 2 = \frac{1+14}{7} = \frac{15}{7}$$

$$t_2 = \frac{2}{7} + 2 = \frac{14+2}{7} = \frac{16}{7}$$

$$\therefore d = t_2 - t_1 = \frac{16}{7} - \frac{15}{7} = 1/7$$

$$\boxed{a = \frac{15}{7}}$$

$$S_{35} = \frac{n}{2} [2a + (n-1)d]$$

$$S_{35} = \frac{35}{2} [2(\frac{15}{7}) + (35-1)(\frac{1}{7})]$$

$$\begin{aligned}
 S_{35} &= \frac{35}{2} \left[\frac{30}{7} + \frac{364}{7} \right] = 0 \\
 &= \frac{35}{2} \left[\frac{394}{7} \right] = \\
 &= 5 \times 38 \\
 &= 190
 \end{aligned}$$

29) In an A.P the 3rd term is 4 times the 1st term and 6th term is 17
 1st find the series

$$t_3 = a + 2d = 4(t_1) = 4a$$

$$t_6 = a + 5d = 17$$

$$2d = 4a - a$$

$$= 3a$$

$$d = \frac{3}{2}a \quad \text{--- (2)}$$

$$t_6 = 17$$

$$a + 5d = 17$$

$$a + \frac{15}{2}(a) = 17$$

$$\frac{2a + 15a}{2} = 17 \Rightarrow 17a = 34 \Rightarrow a = 2$$

$$d = \frac{3a}{2} = \frac{3(2)}{2} = 3$$

$$t_1 = a = 2$$

$$t_2 = t_1 + d = 2 + 3 = 5$$

$$t_3 = t_2 + d = 5 + 3 = 8$$

30) If the 4th term is 10 times the 1st term and the 8th term is 16 44 find the series

$$t_4 = 10(t_1) = 10a \quad \text{--- ①}$$

$$t_4 = a + 3d = 10a$$

$$3d = 9a$$

$$d = 3a \quad \text{--- ②}$$

$$t_8 = 44$$

$$a + 7d = 44$$

$$a + 7(3a) = 44$$

$$a + 21a = 44$$

$$22a = 44$$

$$a = 2$$

Sub a in ②

$$d = 3(2) = 6$$

$$t_1 = a = 2$$

$$t_2 = t_1 + d = 2 + 6 = 8$$

$$t_3 = t_2 + d = 8 + 6 = 14$$

31) The 4th term is 7 times the 1st term and the 14th term is

81 Find the series and 25th term

$$t_4 = 7(t_1) = 7a \quad \text{--- ①}$$

$$t_4 = a + 3d = 7a$$

$$3d = 6a$$

$$d = 2a \quad \text{--- ②}$$

$$t_{14} = 81$$

$$a + 13d = 81$$

$$a + 13(2a) = 81 \Rightarrow a + 26a = 81 \Rightarrow a = 3$$

$$d = 2a \Rightarrow d = 2(3) \Rightarrow d = 6$$

$$t_1 = a = 3$$

$$t_2 = t_1 + d = 3 + 6 = 9$$

$$\begin{aligned} t_4 &= 21a \\ a + 3d &= 21a \\ 3d &= 20a \\ d &= \frac{20}{3}a \end{aligned}$$

$$t_3 = t_2 + d = 9 + 6 = 15$$

∴ The series are 3, 9, 15

$$\begin{aligned} t_{25} &= a + (n-1)d \\ &= 3 + (25-1)d \\ &= 3 + (24)6 \\ &= 147 \end{aligned}$$

✓ 32) If the 4th term is 4 times the 1st term and 15th term is 52.5 find the series & S₁₇.

$$t_4 = 4(t_1) = 4a \quad \text{--- (1)}$$

$$t_4 = a + 3d = 4a$$

$$t_4 = 3d = 3a$$

$$d = a \quad \text{--- (2)}$$

$$t_{15} = 52.5$$

$$a + 14d = 52.5$$

$$t_1 = a = 3.5$$

$$a + 14(a) = 52.5$$

$$15a = 52.5$$

$$a = 3.5$$

$$t_2 = t_1 + d = 3.5 + 3.5 = 7$$

$$t_3 = t_2 + d = 7 + 3.5 = 10.5$$

∴ series are 3.5, 7, 10.5

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{17} = \frac{n}{2} [2(3.5) + 16(3.5)]$$

$$= \frac{17}{2} [7 + 56]$$

$$= \frac{17(63)}{2} = 535.5$$

33) The 3rd term is 5 times 1st term. and $S_{25} = 1875$. Find series

Sol: $t_3 = 5t_1 = 5(a) = 5a$ — (1)

$$a + 2d = 5a$$

18) $3d = 5a - a = 4a \Rightarrow d = \frac{4}{3}a$ — (2)

$$S_{25} = S_n = \frac{n}{2} [2a + (n-1)d] \Rightarrow 1875 = \frac{25}{2} [2a + 24(\frac{4}{3}a)]$$

$$1825 = \frac{25}{2} [2a + 24(\frac{4}{3}a)] \quad 3750 = 25 [2a + 48a]$$

$$3680 = 25 [2a + 48a]$$

$$\frac{50a}{24a} = 146$$

$$a = 146/50$$

$$150 = 50a \Rightarrow a = 3$$

$$\therefore d = 2(a) = 2(3) = 6$$

$$\therefore t_1 = a = 3,$$

$$\therefore t_2 = a + d = 3 + 6 = 9.$$

$$\therefore t_3 = a + 2d = 3 + 2(6) = 3 + 12 = 15$$

34) The sum of the 3 numbers of A.P is 15 and sum of the square is 83 let the 3 numbers be $a-d, a, a+d$

Sol: Given sum = 15

$$\Rightarrow a - d + a + a + d = 15$$

$$\rightarrow 3a = 15$$

$$a = 5$$

Given sum of the square = 83

$$\Rightarrow (a-d)^2 + a^2 + (a+d)^2 = 83$$

$$\Rightarrow a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 83$$

$$3a^2 + 2d^2 = 83$$

$$3(5)^2 + 2d^2 = 83$$

$$2d^2 = 83 - 75$$

$$2d^2 = 8$$

$$d^2 = 4$$

$$d = \pm 2$$

Case 1 : $a=5, d=2$

$$a-d=5-2=3$$

$$a=5$$

$$a+d=5+2=7$$

Case 2 : $a=5, d=-2$

$$a-d=5-(-2)=7$$

$$a=5$$

$$a+d=5-2=3$$

35) Sum of the 3 numbers is 12 & sum of their squares is 66

Sol:- $a-d, a, a+d$

$$\text{Sum} = 12$$

$$a-d+a+a+d=12$$

$$3a=12 \Rightarrow a=4$$

Given Sum of the squares = 66

$$(a-d)^2 + a^2 + (a+d)^2 = 66$$

$$3a^2 + 2d^2 = 66$$

$$3(4)^2 + 2d^2 = 66$$

$$48 + 2d^2 = 66$$

$$2d^2 = 18$$

$$d^2 = 9$$

$$d = \pm 3$$

Case 1 : $a=4, d=3$

$$a-d=1$$

$$a=4$$

$$a+d=7$$

Case 2 : $a=4, d=-3$

$$a-d=7$$

$$a=4$$

$$a+d=1$$

✓ 36) Sum of the 3 numbers is 12 & sum of their squares is 56

Given Sum = 12

$$a-d+a+a+d=12$$

$$3a=12$$

$$a=4$$

Given, Sum of the squares = 56

$$(a-d)^2 + a^2 + (a+d)^2 = 56$$

$$a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 56$$

$$3a^2 + 2d^2 = 56$$

$$3(4)^2 + 2d^2 = 56$$

$$48 + 2d = 56$$

$$\Rightarrow 2d = 56 - 48$$

$$\Rightarrow d = 4$$

$$d = \pm 4$$

Case 1: $a=4, d=2$

$$a-d = 4-2=2$$

$$a=4$$

$$a+d = 4+2=6$$

Case 2: $a=4, d=-2$

$$a-d = 4+2=6$$

$$a=4$$

$$a+d = 4-2=2$$

37) Sum of the 3 numbers is 15 and Sum of their product is 30

$$\text{Sum} = 15$$

$$(a-d) + a + a+d = 15$$

$$3a = 15$$

$$a = 5$$

Given Sum of their product = 80

$$(a-d)(a)(a+d) = 80$$

$$(a^2 - d^2)a = 80$$

$$25 - d^2(5) = 80$$

$$125 - 5d^2 = 80$$

$$d^2 = 9$$

$$d = \pm 3$$

Case 1: $a=5, d=3$

$$a-d = 5-3$$

$$a=5$$

$$a+d = 5+3=8$$

Case 2:

$$a=5, d=-3$$

$$a-d = 5+3$$

$$= 8$$

$$a+d = 5-3$$

$$= 2$$

38) Sum is 15, product = 105

$$\text{Sum} = 15$$

$$(a-d) + a + a + d = 15$$

$$3a = 15$$

$$a = 5$$

Given Sum of the products = 105

$$(a-d)(a)(a+d) = 105$$

$$(a-d)(a+d)(a) = 105$$

$$(a^2 - d^2)a = 105$$

$$(5^2 - d^2)5 = 105$$

$$25 - 5d^2 = 105$$

$$-5d^2 = +20$$

$$d^2 = 4$$

$$d = \pm 2$$

Case 1: $a=5, d=2$

$$a-d = 5-2 = 3$$

$$a = 5$$

$$a+d = 5+2 = 7$$

Case 2: $a=5, d=-2$

$$a-d = 5+2 = 7$$

$$a = 5$$

$$a+d = 5-2 = 3$$

39) Sum is 21, product 315

$$a-d + a + a + d = 21$$

$$3a = 21$$

$$a = 7$$

Product of their sums = 315

$$(a-d)(a+d)(a) = 315$$

$$(a^2 - d^2)(a) = 315$$

$$(7^2 - d^2)7 = 315$$

$$(49 - d^2)7 = 315$$

$$343 - 7d^3 = 315$$

$$-7d^3 = -28$$

$$d^3 = 4$$

$$d = \pm 2$$

Case 1

$$a = 7, d = 2$$

$$a - d = 7 - 2 = 5$$

$$a + d = 7 + 2 = 9$$

Case 2

$$a = 7, d = -2$$

$$a - d = 7 - (-2)$$

$$= 9$$

$$a = 7$$

$$a + d = 7 - 2 = 5$$

$$\begin{array}{r} 9 \\ 71 \\ \hline 31 \end{array}$$

40) Sum is 21, product is 231

$$a - d + a + a + d = 21$$

$$3a = 21$$

$$a = 7$$

Product of three sums = 231

$$(a - d)(a + d)(a) = 231$$

$$(a^3 - d^3)a = 231$$

$$(7^3 - d^3)7 = 231$$

$$(49 - d^3)7 = 231$$

$$-7d^3 = 231 - 343$$

$$-7d^3 = -112$$

$$d^3 = 16 \Rightarrow d = \pm 4$$

Case 1:

$$a = 7, d = 4$$

$$a - d = 7 - 4 = 3, a = 7$$

$$a + d = 7 + 4 = 11$$

Case 2: $a = 7, d = -4$

$$a - d = 7 - (-4) = 11$$

$$a = 7$$

$$a + d = 7 - 4 = 3$$

5/7/18

41) Sum of the 4 numbers is 20 & sum of their squares is 120 / Product 384

let the 4 numbers be $a-3d, a-d, a+d, a+3d$

Given Sum = 20

$$a-3d + a-d + a+d + a+3d = 20$$

$$4a = 20$$

$$a = 5$$

Given Sum of the squares is 120

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$$

$$a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 120$$

$$4a^2 + 20d^2 = 120$$

$$4(25) + 20d^2 = 120$$

$$20d^2 = 20$$

$$d^2 = 1$$

$$d = \pm 1$$

$$(a^2 - 9d^2)(a^2 - d^2) = 384$$

$$a^4 - 10a^2d^2 + 9d^4 = 384$$

$$5^4 - 10(5)^2d^2 + 9d^4 = 384$$

$$625 - 250d^2 + 9d^4 = 384$$

$$9d^4 - 250d^2 + 241 = 0$$

$$9d^4 - 9d^2 - 241d^2 + 241 = 0$$

$$9d^2(d^2 - 1) - 241(d^2 - 1) = 0$$

$$d^2 - 1 = \frac{241}{9} \times$$

$$d^2 = 1$$

$$d = \pm 1$$

Case 1: $d=1, a=5$

$$a-3d = 5-3(1) = 2$$

$$a-d = 5-1 = 4$$

$$a+d = 5+1 = 6$$

$$a+3d = 5+3(1) = 8$$

Case 2: $d=-1, a=5$

$$a-3d = 5-3(-1)$$

$$= 5+3 = 8$$

$$a-d = 5-(-1) = 6$$

$$a+d = 5+(-1) = 4$$

$$a+3d = 5+3(-1) = 2$$

42) Sum of the 4 numbers is 24 sum of their squares is 164

let the 4 numbers be $a-3d, a-d, a+d, a+3d$

Given Sum = 24

$$a-3d + a-d + a+d + a+3d = 24$$

$$4a = 24$$

$$a = 6$$

Given Sum of the squares is 164

$$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 164$$

$$a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 164$$

$$4a^2 + 20d^2 = 164$$

$$4(30) + 20d^{\vee} = 164$$

$$20d^{\vee} = 164 - 144$$

$$20d^{\vee} = 20$$

$$d^{\vee} = 1$$

$$d = \pm 1$$

Case 1: $a = 6, d = 1$

$$a - 3d = 6 - 3(1) = 3$$

$$a - d = 6 - 1 = 5$$

$$a + 3d = 6 + 3(1) = 9$$

$$a + d = 6 + 1 = 7$$

Case 2: $a = 6, d = -1$

$$a - 3d = 6 - 3(-1) = 9$$

$$a - d = 6 - (-1) = 7$$

$$a + d = 6 + (-1) = 5$$

$$a + 3d = 6 + 3(-1) = 3$$

43) Sum of 4 numbers is 40 & Product of their first & last term is 91
*
22

let the 4 numbers $a-3d, a-d, a+d, a+3d$

$$a-3d + a-d + a+d + a+3d = 40$$

$$4a = 40$$

$$a = 10$$

$$(a-3d)(a+3d) = 91$$

$$a^2 + 3ad - 3ad - 9d^2 = 91$$

$$a^2 - 9d^2 = 91$$

$$10^2 - 9d^2 = 91$$

$$d^2 = 1$$

$$d = \pm 1$$

Case 1:

$$a - 3d = 10 - 3(1) = 7$$

$$a - d = 10 - 1 = 9$$

$$a + d = 10 + 1 = 11$$

$$a + 3d = 10 + 3(1) = 13$$

Case 2:

$$a - 3d = 10 - 3(-1) = 13$$

$$a - d = 10 - (-1) = 11$$

$$a + d = 10 + (-1) = 9$$

$$a + 3d = 10 + 3(-1) = 7$$

44) Sum of the 4 numbers is 24 & their product is 945.

let the 4 numbers $a-3d, a-d, a+d, a+3d$

$$a-3d + a-d + a+d + a+3d = 24$$

$$4a = 24$$

$$a = 6$$

$$\text{product } (a-3d)(a-d)(a+d)(a+3d) = 945.$$

$$(a-3d)(a-d)(a+d)(a+3d) = 945$$

$$(-36-3d)(36-d) = 945.$$

$$1296 - 36d - 108d + 3d^2 = 945$$

$$36[36-d-3d+3d^2] \times$$

$$\Rightarrow a^4 - a^2d^2 - 9a^2d^2 + 9d^4 = 945$$

$$\Rightarrow 6^4 - 10 \cdot 6^2 d^2 + 9d^4 = 945$$

$$\Rightarrow 1296 - 360d^2 + 9d^4 = 945$$

$$\Rightarrow -360d^2 + 9d^4 = -351$$

$$\Rightarrow -360d^2 + 9d^4 + 351 = 0$$

$$\Rightarrow 9d^4 - 360d^2 + 351 = 0$$

$$\Rightarrow 9[d^4 - 40d^2 + 39] = 0$$

$$d^4 - 40d^2 + 39 = 0$$

$$\Rightarrow d^4 - 39d^2 + d^2 + 39 = 0$$

$$\Rightarrow d^2[d^2 - 39] - 1[d^2 - 39] = 0$$

$$\Rightarrow (d^2 - 39)(d^2 - 1) = 0$$

$$d^2 = 39, 1$$

$$d = \pm 39, \pm 1$$

$$\begin{array}{r} 13 \overline{) 39} \\ \underline{39} \\ 0 \end{array}$$

Case 1 : $a=6, d=1$

$$a-3d = 6-3(1) = 3$$

$$a-d = 6-1 = 5$$

$$a+d = 6+1 = 7$$

$$a+3d = 6+3(1) = 9$$

Case 2 : $a=6, d=-1$

$$a-3d = 6-3(-1) = 9$$

$$a-d = 6+1 = 7$$

$$a+d = 6-1 = 5$$

$$a+3d = 6+3(-1) = 3$$

45) Sum of the 4 numbers is 32 and their product is ~~3465~~ ¹⁶⁸⁰

let the numbers $a-3d, a-d, a+d, a+3d$

$$\text{Sum} = a-3d + a-d + a+d + a+3d = 32$$

$$4a = 32$$

$$a = 8$$

$$\text{Product} = (a-3d)(a-d)(a+d)(a+3d) = ~~3465~~ ¹⁶⁸⁰$$

$$(a^2 - 9d^2)(a^2 - d^2) = ~~3465~~ ¹⁶⁸⁰$$

$$a^4 - a^2d^2 - 9a^2d^2 + 9d^4 = ~~3465~~ ¹⁶⁸⁰$$

$$8^4 - 8^2 \cdot 10 \cdot d^2 + 9d^4 = ~~3465~~ ¹⁶⁸⁰$$

$$4096 - 640d^2 + 9d^4 = ~~3465~~ ¹⁶⁸⁰$$

$$9d^4 - 640d^2 = -631$$

$$9d^4 - 640d^2 + 631 = 0$$

$$= \frac{-(-640) \pm \sqrt{(640)^2 - 4(9)(631)}}{2(9)}$$

$$= \frac{640 \pm \sqrt{409600 - 22716}}{18} = \frac{640 \pm 622}{18}$$

$$\begin{matrix} 409600 \\ - 22716 \\ \hline \end{matrix}$$

2, 6, 10, 14

(1, 3, 5, 7) ^{prod 105}

$$\text{Case 1: } \frac{640 \pm 622}{18}$$

$$= 6 \pm 0.11$$

$$\text{Case 2: } \frac{640 - 622}{18}$$

$$= \frac{18}{18} = 1$$

$$\text{Case 1: } a=8, d=1$$

$$a-3d = 8-3(1) = 15$$

$$a-d = 8-1 = 7$$

$$a+d = 8+1 = 9$$

$$a+3d = 8+3(1) = 11$$

$$\text{Case 2: } a=8, d=-1$$

$$a-3d = 8-3(-1) = 8+3 = 11$$

$$a-d = 8-(-1) = 9$$

$$a+3d = 8+3(-1) = 5$$

$$a+d = 8+(-1) = 7$$

6/7/18

46) ✓

*

24

Insert 4 A.M between 5 and 20

$$t_1 = a = 5$$

$$t_6 = 20, a+5d = 20$$

$$5d = 20-5$$

$$5d = 15$$

$$d = 3$$

$$t_1 = a = 5$$

$$t_2 = t_1 + d = 5+3 = 8$$

$$t_3 = t_2 + d = 8+3 = 11$$

$$t_4 = t_3 + d = 11+3 = 14$$

$$t_5 = t_4 + d = 14+3 = 17$$

$$t_6 = t_5 + d = 17+3 = 20$$

47) Insert 5 A.M between 9 & 21

$$t_1 = a = 9$$

$$t_2 + t_7 = 21 = a+6d = 21$$

$$6d = 21-9$$

$$6d = 12$$

$$d = 2$$

$$t_1 = a = 9$$

$$t_2 = a+d = 9+2 = 11$$

$$9 \quad \quad \quad 21$$

$$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7$$

$$t_3 = t_2 + d = 11 + 2 = 13$$

$$t_4 = t_3 + d = 13 + 2 = 15$$

$$t_5 = t_4 + d = 15 + 2 = 17$$

$$t_6 = t_5 + d = 17 + 2 = 19$$

$$t_7 = t_6 + d = 19 + 2 = 21$$

48) G. A.M b/w 2 & 32

$$\begin{array}{ccccccc} 2 & - & - & - & - & - & 32 \\ t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \end{array}$$

$$t_1 = a = 2$$

$$d = t_8 - t_1 = 32 - 2 = 30$$

$$7d = 30$$

$$d = 30/7$$

$$t_1 = a = 2$$

$$t_2 = a + d = 2 + \frac{30}{7} = \frac{14 + 30}{7} = \frac{44}{7}$$

$$t_3 = t_2 + d = \frac{44}{7} + \frac{30}{7} = \frac{74}{7}$$

$$t_4 = t_3 + d = \frac{74}{7} + \frac{30}{7} = \frac{104}{7}$$

$$t_5 = t_4 + d = \frac{104}{7} + \frac{30}{7} = \frac{134}{7}$$

$$t_6 = t_5 + d = \frac{134}{7} + \frac{30}{7} = \frac{164}{7}$$

$$t_7 = t_6 + d = \frac{164}{7} + \frac{30}{7} = \frac{194}{7}$$

$$t_8 = t_7 + d = \frac{194}{7} + \frac{30}{7} = \frac{224}{7}$$

49)

$$t_2 = 34, t_7 = 48$$

$$\begin{array}{ccccccc} - & \frac{34}{t_2} & - & - & - & - & \frac{48}{t_7} & - \\ & & & & & & & t_8 \end{array}$$

$$t_7 = 48 = a + 6d = 48$$

$$t_2 = 34 = a + d = 34$$

$$\begin{array}{r} (-2) \quad (-) \quad (-) \\ \hline 5d = 14 \end{array}$$

$$d = 14/5$$

$$t_2 = a + \frac{14}{5} = 34$$

$$\Rightarrow a = 34 - \frac{14}{5} = \frac{156}{5}$$

$$49) t_1 = a = \frac{156}{5}$$

$$t_2 = a + d = \frac{156}{5} + \frac{14}{5} = \frac{170}{5}$$

$$t_3 = a + 2d = \frac{156}{5} + \frac{28}{5} = \frac{184}{5}$$

$$t_4 = a + 3d = \frac{156}{5} + \frac{42}{5} = \frac{198}{5}$$

$$t_5 = a + 4d = \frac{156}{5} + \frac{56}{5} = \frac{212}{5}$$

$$t_6 = a + 5d = \frac{156}{5} + \frac{70}{5} = \frac{226}{5}$$

$$t_7 = 48$$

$$t_8 = a + 7d = \frac{156}{5} + 7\left(\frac{14}{5}\right) = \frac{156}{5} + \frac{98}{5} = \frac{254}{5}$$

* Formula:

$$1) n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

2)

i) What is the sum of the integers from 1 to 100 that are divisible by 2 or 3, not divisible by 2 or 3.

sol Given $S = 1 + 2 + \dots + 100$

$$\therefore S = \frac{n(n+1)}{2} = \frac{100(100+1)}{2} = 5050$$

— ①

$$S(S) = 5050$$

let $A = \text{Multiple of } 2$

$$A = 2 + 4 + \dots + 100$$

$$A = \frac{t_n - t_1}{A} + 1 = \frac{100 - 2}{2} + 1 = 49 + 1 = 50 \Rightarrow n(A) = 50$$

$$2) \text{ let } A = 2 + 4 + \dots + 100$$

$$= 2 [1 + 2 + \dots + 50]$$

$$S(A) = 2 \left[\frac{50(50+1)}{2} \right] = 2550$$

let B = multiple of 3

$$B = 3 + 6 + \dots + 99$$

$$B = \frac{t_n - t_1}{d} = \frac{99 - 3}{3} + 1 = \frac{96}{3} + 1 = 32 + 1 = 33$$

$$\Rightarrow n(B) = 33$$

$$A = 3 + 6 + \dots + 99$$

$$= 3 [1 + 2 + \dots + 33]$$

$$S(B) = 3 \left[\frac{33(33+1)}{2} \right] = \frac{(1122)3}{2} = 1683$$

2) $A \cap B$ = Simultaneous occurrence of A & B

$\therefore A \cap B$ = multiples of $2(3) = 6$

$$\therefore A \cap B = 6 + 12 + \dots + 96$$

$$n(A \cap B) = \frac{t_n - t_1}{d} + 1$$

$$= \frac{96 - 6}{6} + 1 = 16$$

$$S(A \cap B) = 6 [1 + 2 + \dots + 16]$$

$$= \frac{n(n+1)}{2} = \frac{16(16+1)}{2} = \frac{272}{2} = 136$$

$$= 16(8)(17) = 816$$

Sum of the numbers which are divisible by 2 or 3 = $S(A \cup B)$

$$S(A) = 2550 + 1683 - 816$$

$$= 4233 - 816 = 3417$$

Not divisible by 2 or 3

$$\therefore S(A \cup B)' = S(S) - S(A \cap B)$$

$$= 85050 - 3417$$

$$= 1633$$

✓ (2) Sum of the numbers between 1 to 200 that are divisible by 3 or 5 (not divisible by 3 or 5.)

Sol Given $S = 3 + 6 + \dots + 198$

$$S = 1 + 2 + \dots + 200$$

$$\therefore S = \frac{n(n+1)}{2} = \frac{200(200+1)}{2} = \frac{200(201)}{2} = 20100 \quad \text{--- (1)}$$

$$n(S) = 20100$$

let $A = \text{multiple of } 3$

$$A = 3 + 6 + \dots + 198$$

$$A = \frac{t_n - t_1}{A} + 1$$

$$= \frac{198 - 3}{3} = \frac{195}{3} = 65 + 1 = 66$$

$$n(A) = 66$$

$$\text{let } A = 3 + 6 + \dots + 198$$

$$= 3 [1 + 2 + \dots + 66]$$

$$S(A) = 3 \left[\frac{66(66+1)}{2} \right] = 6633$$

let $B = \text{multiple of } 5$

$$B = 5 + 10 + \dots + 200$$

$$B = \frac{t_n - t_1}{B} + 1 = \frac{200 - 5}{5} + 1 = \frac{195}{5} + 1 = 39 + 1 = 40$$

$$n(B) = 40$$

$$A = 5 + 10 + \dots + 200$$

$$= 5 [1 + 2 + \dots + 40]$$

$$S(B) = 5 \left[\frac{40(40+1)}{2} \right]$$

$$= 5 \left[\frac{40(41)}{2} \right]$$

$$= 4100$$

$A \cap B$ = Simultaneous occurrence of A & B

$$\therefore A \cap B = \text{multiple of } n(3) = 15$$

$$\therefore A \cap B = 15 + 30 + \dots + 195$$

$$n(A \cap B) = \frac{195 - 15}{15} + 1 = 13 + 1 = 14$$

$$S(A \cap B) = 15 [1 + 2 + \dots + 13]$$

$$= \frac{n(n+1)}{2} \times 15$$

$$= 15 \left[\frac{13(13+1)}{2} \right] = 1365$$

Sum of the numbers which are divisible by 3 or 5 = $S(A \cup B)$

$$S(A) = 4006633 + 4100 - 1365$$

$$= 9368$$

Q. 1112
3) What is the sum of the integers from 1 to 100 that are divisible by 7.

Given $S = 7 + 14 + \dots + 98$

$$\therefore S = \frac{n(n+1)}{2} = \frac{98(98+1)}{2} =$$

Sol Given $S = 1 + 2 + \dots + 100$

$$S = \frac{n(n+1)}{2} = \frac{100(100+1)}{2} = 5050$$

$$n(S) = 5050$$

let $A =$ multiplier of 7.

$$A = 7 + 14 + \dots + 98$$

$$= \frac{t_n - t_1}{n} + 1$$

$$= \frac{98 - 7}{7} + 1 = \frac{91}{7} + 1 = 14$$

$$S(A) = 7 + 14 + 21 + \dots + 98$$

$$= 7[1 + 2 + \dots + 14]$$

$$S(A) = 7 \left[\frac{14(14+1)}{2} \right]$$

$$= 7 \left[\frac{210}{2} \right] = 735$$

Sum of the numbers between 1 to 100 that are divisible by 7 are 735

$\therefore$ Sum of the numbers between 1 to 100 that are not divisible by 7

$$S(A) = S(S) - S(A)$$

$$735 = 5050 - 4315$$

$$5785 = 5050 - 4315 \Rightarrow 1470$$

4) Find the sum of all integers from 1 to 1000 that are not divisible by 9

sol

$$S = 1 + 2 + \dots + 1000$$

$$n(s) = \frac{1000(1000+1)}{2}$$

$$= 500500$$

let A = Multiplier of 9

$$A = 9 + 18 + \dots + 999$$

$$= \frac{tn - t1}{2} + 1$$

$$= \frac{999 - 9}{9} + 1 = 111$$

$$S(A) = 9 + 18 + \dots + 999$$

$$= 9[1 + 2 + \dots + 111]$$

$$S(A) = 9 \left[\frac{111(111+1)}{2} \right]$$

$$= 9 \left[\frac{111(112)}{2} \right] = 55944$$

Sum of the numbers between 1 to 1000 that are divisible by

9 are 55944

$\therefore$ Sum of the numbers between 1 to 1000 that are not divisible by 9

$$S(A) = S(s) - S(A)$$

$$= 500500 - 55944$$

$$= 444556 //$$

$$\begin{array}{r} 444556 \\ - 55944 \\ \hline 444556 \end{array}$$

26) Find the sum of all the integers from 500 to 1000 that are exactly divisible by 1) 11
2) 25.

Sol let A = Multiple of 11 between 500 to 1000

$$= 506 + 517 + \dots + 990$$

$$n(A) = \frac{990 - 506}{11} + 1 =$$

$$= 45$$

$$S(A) = \frac{45}{2} [506 + 990] = 33660$$

let B = Multiple of 25 between 500 to 1000

$$= 500 + 525 + \dots + 1000$$

$$n(B) = \frac{1000 - 500 + 1}{25} = \frac{500 + 1}{25} = 20 + 1 = 21$$

$$S(B) = \frac{21}{2} [500 + 1000]$$

$$= 15750$$

✓ 27) How many numbers are there between 74 & 25556 which are exactly divisible by 1) 15, 9, 50

$$\text{let } A = 75 + 90 + \dots + 25545$$

$$n(A) = \frac{25545 - 75 + 1}{15} = 1698 + 1 = 1699$$

$$\text{let } B = 81 + 90 + \dots + 25551$$

$$n(B) = \frac{25551 - 81 + 1}{9} = 2830 + 1 = 2831$$

$$\text{let } C = 100 + 150 + \dots + 2550$$

$$n(C) = \frac{2550 - 100}{50} + 1 = 509 + 1 = 510$$

7) The first term of AP is 14. The sum of the first 5 terms is same as the first 10 terms but opposite in sign. Find t_3

$$\text{Given } t_1 = a = 14$$

$$S_5 = -S_{10}$$

$$\frac{5}{2} [2(14) + (5-1)d] = -\frac{10}{2} [2(14) + (10-1)d]$$

$$5[28 + 4d] = -10[28 + 9d]$$

$$28 + 4d = -56 - 18d$$

$$28 + 56 = -18d - 4d$$

$$84 = -22d$$

$$d = -\frac{84}{22} = -\frac{42}{11}$$

$$= -42/11$$

$$\begin{aligned} t_3 &= a + (n-1)d \\ &= 14 + (2) \cdot \frac{-42}{11} \\ &= 14 + \frac{(-84)}{11} \\ &= \frac{154 - 84}{11} \end{aligned}$$

$$\begin{aligned} t_3 &= a + 2d \\ &= 14 + 2 \left(\frac{-42}{11} \right) \\ &= 14 - \frac{84}{11} \\ &= \frac{154 - 84}{11} = \frac{70}{11} \end{aligned}$$

12/7/18
 8) A man repays a loan of ₹ 3250 by installments he pays ₹ 20 in the first month & then increases it every month. In how many months does he repay the loan amt.

29 Total loan Amt = $S_n = 3250$

1st installment = $t_1 = a = 20$

$\therefore d = 15$

K.K.T $S_n = \frac{n}{2} [2a + (n-1)d]$

$$3250 = \frac{n}{2} [40 + (n-1)15]$$

$$3250 = \frac{n}{2} [40 + 15n - 15]$$

$$6500 = 15n^2 + 40n - 15n$$

$$6500 = 15n^2 + 25n$$

$$250 \Rightarrow 25n + 15n^2 - 6500 = 0$$

$$\Rightarrow 5(5n + 3n^2 - 1300) = 0$$

$$\Rightarrow 3n^2 + 5n - 1300 = 0$$

$$\Rightarrow \frac{-5 \pm \sqrt{25 - 4(-1300)(3)}}{2(3)}$$

$$\Rightarrow \frac{-5 \pm \sqrt{25 + 15600}}{6}$$

$$\Rightarrow \frac{-5 \pm \sqrt{125}}{6}$$

$$\Rightarrow \frac{-5 - 125}{6}$$

$$\Rightarrow \frac{-5 + 125}{6}$$

$$\Rightarrow -5 - 16.66$$

$$\Rightarrow -5 + 20$$

$$= -21.66$$

$$\Rightarrow 20.$$

Q A person pays Rs. 975 in monthly installments. The first installment is Rs. 100 and each installment being less than the former by Rs. 5. In how many months does he repay entire sum

sol $S(n) = 975$, $t_1 = a = 100$, $d = -5$

We know that, $S(n) = \frac{n}{2} [2a + (n-1)d]$

$$975 = \frac{n}{2} [2(100) + (n-1)(-5)]$$

$$975 = \frac{n}{2} [200 + n - 5n]$$

$$975 = \frac{n}{2} [200 - 4n]$$

$$1950 = 200n - 4n^2$$

$$-4n^2 + 200n - 1950 = 0$$

$$\Rightarrow 4n^2 - 200n + 1950 = 0$$

$$\Rightarrow 4n^2 - 41n + 390 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-41) \pm \sqrt{(-41)^2 - 4(1)(390)}}{2(1)}$$

$$= \frac{41 \pm \sqrt{1681 - 1560}}{2}$$

$$= \frac{41 \pm 11}{2}$$

Case 1

$$\frac{41 + 11}{2} = \frac{52}{2}$$

$$= 26$$

Case 2

$$\frac{41 - 11}{2} = \frac{30}{2}$$

$$= 15$$

10) A Club Consist of members whose ages are in Ap Common difference being 3 months the youngest member of the club is 4yrs old and the sum of the ages of all the members is 250 Find the members

sol $S_n = \frac{n}{2} [2a + (n-1)d]$

$$250 = \frac{n}{2} [14 + \frac{1}{4}n - \frac{1}{4}]$$

$$= \frac{n}{2} \left[\frac{56+n-1}{4} \right]$$

$$500 = n \left[\frac{55+n}{4} \right]$$

$$2000 = 55n + n^2$$

$$\Rightarrow n^2 + 55n - 2000 = 0$$

$$\Rightarrow n^2 + 80n - 25n - 2000 = 0$$

$$\Rightarrow n(n-25) + 80(n-25) = 0$$

$$\Rightarrow (n-25)(n+80) = 0$$

$$\Rightarrow n = 25, -80$$

32 ✓

(11) $1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots$

sol 1st factor : 1, 2, 3 - - - -

$$a=1, d=2-1=1$$

$$d=1$$

$$t_n = a + (n-1)d$$

$$= 1 + (n-1)1$$

$$= n \quad \boxed{\text{i.e; } t_n = 1}$$

2nd factor : 3, 5, 7 - - -

$$a=3, d=2$$

$$t_n = a + (n-1)d$$

$$= 3 + (n-1)2$$

$$= 3 + 2n - 2$$

$$= 2n + 1$$

$$\therefore t_n = 2(2n+1) = 2n^2 + 2$$

Apply sum on both sides

$$S_n = \sum (2n^2 + n)$$

$$= 2\sum n^2 + \sum n$$

$$= 2 \left[\frac{n(n+1)(2n+1)}{6} \right] + \left[\frac{3(n)(n+1)}{3 \times 2} \right]$$

$$= 2 \left[\frac{n(n+1)(2n+1) + 3n(n+1)}{6} \right]$$

$$= \frac{n(n+1)}{6} [2(2n+1) + 3]$$

$$= \frac{n(n+1)(4n+2+3)}{6}$$

$$= \frac{n(n+1)(4n+5)}{6}$$

✓
⑫ $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots$

Ist factor: $1, 2, 3, \dots$

$$a=1, d=1$$

$$t_n = 1 + (n-1)1$$

$$= n \quad [\because t_n = n]$$

Padmasree

II factor: $2, 3, 4, \dots$

$$a=2, d=1$$

$$t_n = 2 + (n-1)1$$

$$= 2 + n - 1$$

$$t_n = n + 1$$

III factor: $3, 4, 5, \dots$

$$a=3, d=1$$

$$t_n = 3 + (n-1)1$$

$$= 3 + n - 1$$

$$t_n = n + 2$$

$$t_n = t_{n1} \times t_{n2} \times t_{n3}$$

$$= n(n+1)(n+2)$$

$$= (n^3 + n^2)(n+2)$$

$$= n^3 + 2n^2 + n^3 + 2n$$

$$= n^3 + 3n^2 + 2n$$

$$S_n = S_n^3 + 3S_n^2 + 2S_n$$

$$= \sum n^3 + 3 \sum n^2 + 2 \sum n$$

$$= \frac{3(n^2)(n+1)^2}{12} + \frac{(3n)(n+1)2(2n+1)}{6} + \frac{6(2n)(n+1)}{6 \times 2}$$

$$= \frac{3n^2(n+1)^2 + 6n(n+1)(2n+1) + 12n(n+1)}{12}$$

$$= \frac{n(n+1)}{12} [3n(n+1) + 6(2n+1) + 12]$$

$$= \frac{n(n+1)}{12} [3n^2 + 3n + 12n + 6 + 12]$$

$$= \frac{n(n+1)}{12} [3n^2 + 15n + 18]$$

⑬ If $S_n = n(n+2)$ find the series

sol

$$S_1 = t_1 = a = 1(1+2)$$

$$a = 3$$

$$S_2 = t_1 + t_2 = 2(2+2)$$

$$= (4)2 = 8$$

$$\therefore t_2 = S_2 - S_1 = 8 - 3 = 5$$

(or)

$$S_2 = t_1 + t_2$$

$$8 - 3$$

$$8 = 3 + t_2$$

$$t_2 = 8 - 3 = 5$$

$$\text{Now, } d = t_2 - t_1$$

$$= 5 - 3$$

$$d = 2$$

$$t_1 = a = 3$$

$$t_2 = t_1 + d = 3 + 2$$

$$\therefore t_2 = 5$$

$$t_3 = t_2 + d = 5 + 2$$

$$\therefore t_3 = 7$$

$$t_4 = t_3 + d$$

$$= 7 + 2 = 9$$

$\therefore$ The Series is 3, 5, 7, 9 - - -

$$(14) \text{ If } S_n = \frac{n(5n+3)}{2}$$

$$\begin{aligned} \text{Sol } S_1 &= \frac{1(5(1)+3)}{2} \\ &= \frac{1(5+3)}{2} = 4 \end{aligned}$$

$$\therefore S_1 = 4$$

$$\begin{aligned} S_2 &= t_1 + t_2 \\ &= \frac{2(5(2)+3)}{2} = 26/2 = 13 \end{aligned}$$

$$\therefore S_2 = 13$$

$$t_2 = S_2 - S_1$$

$$= 13 - 4$$

$$= 9$$

$$d = 5$$

$\therefore$ The Series are 4, 9, 14, 19.

$$(15) \quad S_n = \frac{n(3n+1)}{2}$$

$$S_1 = \frac{1(3(1)+1)}{2} = 4/2 = 2$$

$$S_2 = \frac{2(3(2)+1)}{2} = \frac{2(7)}{2} = 7$$

$$t_2 = S_2 - S_1$$

$$= 7 - 2 = 5$$

$$\therefore t_2 = 5$$

$$\therefore \text{The series are } t_1 = a = 2$$

$$t_2 = a + d = 2 + 5 = 7$$

$$t_3 = a + 2d = 2 + 10 = 12$$

$$t_4 = a + 3d = 2 + 15 = 17$$

$$\therefore 2, 7, 12, 17, \dots$$

$$(16) \quad 1 \cdot 2 \cdot 5 + 2 \cdot 3 \cdot 7 + 3 \cdot 4 \cdot 9 + \dots$$

$$\text{I factor: } 1, 2, 3, \dots$$

$$a = 1, d = 1$$

$$t_{n1} = 1 + (n-1)1$$

$$= n \therefore t_{n1} = n$$

$$\text{II factor: } 2, 3, 4, \dots$$

$$a = 2, d = 1$$

$$t_{n2} = 2 + (n-1)1$$

$$= 2 + n - 1$$

$$= n + 1$$

$$\text{III factor: } 5, 7, 9, \dots$$

$$a = 5, d = 2$$

$$t_{n3} = 5 + (n-1)2$$

$$= 5 + (2n - 2)$$

$$= 2n + 3$$

Geometric progressions (G.P) :-

$$a, ar, ar^2, ar^3, \dots, ar^{n-1}$$

$$a = t_1$$

$$r = \frac{t_2}{t_1} = \frac{ar}{a} = r$$

$$\therefore t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \text{ if } r < 1$$

$$S_\infty = \frac{a}{1 - r}$$

$$3 \text{ No's} - \frac{a}{r}, a, ar$$

$$4 \text{ No's} - \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$$

* If a, b, c are in G.P that implies $\text{so, } \frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$
 $\Rightarrow b = \sqrt{ac}$

$\therefore b$ is called as Geometric mean of a, c

Hence, the G.M of any two numbers x, y is $\sqrt{xy}$

13/7/18

i) find the 12th term of 3, 6, 12 also find S_{12}

$$t_1 = a = 3, r = 6/3 = 2 > 1$$

$$t_n = a \cdot r^{n-1}$$

$$= 3 \cdot 2^{12-1} = 3 \cdot 2^{11} = 6144$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{3(12^{th} - 1)}{2 - 1} = 3(4096 - 1)$$
$$= 3(4095)$$

$$S_n = 12285$$

2) 2, 4, 8 - - - find t_{10} & S_{10}

$$t_1 = a = 2, r = 4/2 = 2 > 1$$

$$t_n = a \cdot r^{n-1}$$

$$t_{10} = a \cdot r^{n-1} = 2 \cdot 2^9 = 2^{10} = 1024$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^{10} - 1)}{2 - 1} = \frac{2(1024 - 1)}{1}$$

$$= 1023 \times 2$$

$$S_{10} = 2046$$

3) 4, 12, 36 - - - find t_n, S_n, t_{10}, S_{10}

★

$$t_1 = 4, r = 12/4 = 3$$

$$t_n = a \cdot r^{n-1}$$

$$= 4 \cdot 3^{10-1}$$

$$= 4 \cdot 3^9 = 78732$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{4(3^{10} - 1)}{3 - 1} = \frac{4(59049 - 1)}{2}$$

$$= 59048 \times 2$$

$$S_{10} = 118096$$

4) $t_5 = 81, t_2 = 24$ find series & S_{15}

$$t_5 = ar^4 = 81 \text{ --- ①}$$

$$t_2 = ar = 24 \text{ --- ②}$$

$$\frac{\text{①}}{\text{②}} = \frac{ar^4}{ar} = \frac{81}{24}$$

$$\Rightarrow r^3 = \left(\frac{3}{2}\right)^3$$

$$r = 3/2$$

$$ar = 24$$

$$a(3/2) = 24$$

$$a = \frac{24 \times 2}{3}$$

$$a = 16$$

$$t_1 = a = 16$$

$$t_2 = ar = 16 \times 3/2 = 24$$

$$t_3 = ar^2 = 16 \times 9/4 = 36$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{16((3/2)^5 - 1)}{3/2 - 1} = \frac{16((1.5)^5 - 1)}{0.5}$$

$$= \frac{16(437.89 - 1)}{0.5} = 13980.576$$

16/7/18

5) The n^{th} term is 32 8th term is 256. find the series & the

Sum to 8 terms

$$t_n = 32 = ar^4 \quad t_8 = ar^7 = 256 \quad \text{--- ①}$$

$$t_8 = ar^7 = 256 \quad \text{--- ②}$$

$$\frac{\text{①}}{\text{②}} = \frac{ar^4}{ar^7} = \frac{32}{256}$$

$\frac{16 \times 8 \times 2}{128}$
 $\frac{64 \times 32}{128}$
 $\frac{16 \times 8 \times 2}{128}$

$$\Rightarrow \frac{1}{r^3} = \frac{1}{8}$$

$$\Rightarrow 1/r = 1/2 \Rightarrow r = 2$$

$$ar^4 \Rightarrow a(2)^4 = 32 \Rightarrow a = 32/16 \Rightarrow a = 2$$

$$t_1 = 2, t_2 = 4, t_3 = 8$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^8 - 1)}{2 - 1} = \frac{2(256 - 1)}{1} = 2(255) = 510$$

6) first term is 2 and 6th term is 64 find series & sum to 6 terms

Sol $t_1 = a = 2$, $t_6 = ar^5 = 64$ — ②

$$\frac{①}{②} = \frac{a}{ar^5} = \frac{2}{64}$$

$$\Rightarrow \frac{1}{r^5} = \frac{1}{32} \Rightarrow \frac{1}{r} = \frac{1}{2} \Rightarrow r = 2 > 1$$

$$ar^5 = 2(2)^5 = 2(32) = 64 \Rightarrow t_1 = 2, t_2 = 4, t_3 = 8.$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^6 - 1)}{2 - 1} = \frac{2(64 - 1)}{1} = 63(2) = 126$$

7) find the sum of the series $1 + 3 + 9 + \dots$ 15 terms

$$t_1 = a = 1$$

$$r = t_2/t_1 = 3/1 = 3 > 1$$

We know that, $S_n = \frac{a(r^n - 1)}{r - 1}$

$$= \frac{1(3^{15} - 1)}{3 - 1} = \frac{1(3^{15} - 1)}{2}$$

8) find the sum of the series $1, \sqrt{3}, 3, \dots$ S_{12}

*
33

$$t_1 = a = 1$$

$$r = \frac{t_2}{t_1} = \frac{\sqrt{3}}{1} = \sqrt{3} > 1$$

We know that, $S_n = \frac{a(r^n - 1)}{r - 1}$

$$= \frac{1(\sqrt{3}^{12} - 1)}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3}^{12} - 1}{\sqrt{3} - 1}$$

$$= \frac{729 - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{728(\sqrt{3} + 1)}{2} = 364(\sqrt{3} + 1)$$

$$\sqrt{3}^{12} = (3^{1/2})^{12} = 3^6 = 729$$

9) How many terms of GP $-5, 15, -45, \dots$ are taken to make -305 .

sol

$$a = -5$$
$$r = \frac{15}{-5} = -3 < 1$$

$$S_n = -305$$

$$-305 = \frac{a(1-r^n)}{1-r} = \frac{-5(1-(-3)^n)}{1-(-3)}$$

$$\frac{-305}{-5} = \frac{1-(-3)^n}{1+3}$$

$$61 = \frac{1-(-3)^n}{4}$$

$$61(4) = 1-(-3)^n$$

$$(-3)^n = 1-244 = -243$$

$$(-3)^n = (-243)$$

$$(-3)^n = (-3)^5$$

$$n = 5$$

10) find the sum of series $2/3, -1, 3/2, \dots$ to 7 terms

$$a = 2/3$$

$$r = -1/(2/3) = -3/2 < 1$$

$$S_n = \frac{2/3(1-(-3/2)^7)}{1+3/2}$$

$$= \frac{2/3 \left(1 - \frac{(-2187)}{128}\right)}{5/2}$$

$$= \frac{2/3 \left(1 + \frac{2187}{128}\right)}{5/2}$$

$$= \frac{2/3 \left(\frac{128+2187}{128}\right)}{5/2}$$

$$= 2 \times 3 \times 5 \times \frac{2315}{128}$$

$$= \frac{463}{3 \times 32} = \frac{463}{96}$$

11) Find the sum to 10 terms of the series $2, -4, 8, \dots$

$$a = 2$$

$$r = \frac{-4}{2} = -2 < 1$$

$$S_n = \frac{2(1 - (-2)^{10})}{1 - (-2)}$$

$$= \frac{2(1 - 1024)}{3} = \frac{2(-1023)}{3}$$

$$= -682$$

✓ 12) How many terms of GP $-4, 16, -64, \dots$ will make -13108

$$a = -4$$

$$r = \frac{16}{-4} = -4 < 1$$

$$S_n = \frac{-4(1 - (-4)^n)}{1 - (-4)}$$

$$\frac{-13108}{-4} = \frac{1 - (-4)^n}{5}$$

$$3277 \times 5 = 1 - (-4)^n$$

$$16385 = 1 - (-4)^n$$

$$16384 = -(-4)^n$$

$$4^n = 16384$$

$$4^n = 4^7$$

$$\therefore n = 7$$

13) How many terms of the series $-3, +5, -75, \dots$ will make

$$4662$$

$$a = -3$$

$$r = \frac{15}{-3} = -5 < 1$$

$$S_n = \frac{-3(1^0 - (-5)^n)}{1 - (-5)}$$

$$\frac{4662}{-3} = \frac{1^0 - (-5)^n}{1 + 5}$$

$$6(1554) = 1 - (-5)^n$$

$$-9324 - 1 = -(-5)^n$$

$$-9325 = -(-5)^n$$

$$\Rightarrow 5^n = 9325 \Rightarrow$$

$$\frac{42604}{-3} = \frac{-3(1 - (-5)^n)}{6}$$

$$-2604(6) = (1 - (-5)^n)$$

$$-15624 - 1 = -(-5)^n$$

$$-15625 = -(-5)^n$$

$$\text{Let } (-5)^6 = -(-5)^n$$

when bases are equal, powers must be equal

$$\boxed{n = 6}$$

$$\begin{array}{r} 5(9325) \\ 5(1865) \\ 373 \end{array}$$

14) How many terms of the series $4, 12, 36, \dots$ will make 2188.

$$a = 4$$

$$r = 12/4 = 3 > 1$$

$$S_n = \frac{4(1^0 - (-3)^n)}{1 - (-3)}$$

$$2188 = \frac{4(1 - (-3)^n)}{1 + 3}$$

$$4\left(\frac{2188}{4}\right) = 1 - (-3)^n \Rightarrow 1 - (-3)^n = 2188$$

$$3^n = 2187$$

$$n = 7$$

✓
15) which term is $\frac{512}{729}$ of the GP $18, -12, 8, \dots$
✱
35

$$a = 18, r = \frac{-12}{18} = -\frac{2}{3} < 1$$

$$t_n = \frac{512}{729}$$

$$ar^{n-1} = \frac{512}{729}$$

$$18(-\frac{2}{3})^{n-1} = \frac{512}{729}$$

$$(-\frac{2}{3})^{n-1} = \frac{512}{729 \times 18}$$

$$= \frac{256}{6561}$$

$$= \frac{2^8}{3^8}$$

$$= (-\frac{2}{3})^{n-1}$$

$$= (\frac{2}{3})^8 = (-\frac{2}{3})^8$$

$$n-1 = 8$$

$$n = 9$$

18/7/18

16) which term of the series $\frac{1}{128}, \frac{1}{64}, \dots$ is 1
✱

35

$$a = \frac{1}{128}, r = \frac{\frac{1}{64}}{\frac{1}{128}} = \frac{1}{64} \times 128 = 2$$

$$t_n = 1$$

$$510, 20 \text{ is } 5120$$

$$ar^{n-1} = 1$$

$$\frac{1}{128} (2)^{n-1} = 1$$

$$2^{n-1} = 128$$

$$2^n = 256$$

$$2^n = 2^8$$

$$n = 8$$

$$\begin{array}{r} 2 \overline{) 256} \\ 2 \overline{) 128} \\ 2 \overline{) 64} \\ 2 \overline{) 32} \\ 2 \overline{) 16} \\ 2 \overline{) 8} \\ 2 \overline{) 4} \\ 2 \overline{) 2} \end{array} \quad \frac{2}{2}$$

17) The common ratio is 3 & the sum of the n terms is 128 & the last term is 486 find the first term.

$$r = 3 > 1, t_n = 486 = ar^{n-1}$$

$$S_n = 128$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{ar^n - a}{r - 1} = \frac{ar^{n-1} \cdot r - a}{r - 1}$$

$$128 = \frac{(486)(3) - a}{3 - 1}$$

$$(128)2 = 1458 - a$$

$$1456 = 1458 - a$$

$$a = 2$$

$$t_n = 486$$

$$ar^{n-1} = 486$$

$$2(3)^{n-1} = 486$$

$$3^{n-1} = \frac{486}{2} = 243 = 3^5$$

$$n - 1 = 5$$

$$n = 6$$

18) The common ratio is 2 & the last term is 128 & sum of the n terms is 255 find a & n

$$r = 2 > 1, \quad t_n = 128, \quad S_n = 255$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{ar^n - a}{r - 1} = \frac{ar^{n-1} \cdot r - a}{r - 1}$$

$$\Rightarrow 255 = \frac{(128)2 - a}{2 - 1}$$

$$\Rightarrow 255 = 256 - a \Rightarrow a = 1$$

$$t_n = 128$$

$$\Rightarrow ar^{n-1} = 128$$

$$2^{n-1} = 128$$

$$\Rightarrow n - 1 = 7$$

$$n - 1 = 7$$

$$n = 8$$

19) The common ratio is 2 last term is 2048 & sum of the n terms is 4095 find a & n

$$r = 2 > 1$$

$$t_n = 2048, \quad S_n = 4095$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{ar^n - a}{r - 1} = \frac{ar^{n-1} \cdot r - a}{r - 1}$$

$$\Rightarrow 4095 = \frac{a(2 \cdot 2048(2) - a)}{2 - 1} = \frac{4096 - a}{1}$$

$$a = 1$$

$$t_n = 2048$$

$$\Rightarrow ar^{n-1} = 2048$$

$$\Rightarrow (2)^{n-1} = 2048$$

$$n-1 = 2048/2$$

$$n-1 = 2^{11}$$

$$n-1 = 1024$$

$$n = 12$$

✓ 20) The common ratio is 3 the last term is 6561, Sum of the n terms is 9841 find a & n

$$r = 3, t_n = 6561, S_n = 9841$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{ar^n - a}{r - 1} = \frac{ar^{n-1} \cdot r - a}{r - 1}$$

$$\Rightarrow 9841 = \frac{6561(3) - a}{3 - 1}$$

$$\Rightarrow 19682 = 19683 - a$$

$$\Rightarrow a = 1$$

$$t_n = 6561$$

$$ar^{n-1} = 6561$$

$$(3)^{n-1} = 6561$$

$$n-1 = 8$$

$$n = 9$$

$$\begin{array}{r} 3 \overline{) 6561} \\ \underline{2187} \\ 3 \overline{) 729} \\ \underline{243} \\ 3 \overline{) 81} \\ \underline{81} \\ 3 \overline{) 27} \\ \underline{9} \\ 3 \overline{) 3} \\ \underline{3} \\ 0 \end{array}$$

21) Common ratio same as the first term & 5^{th} term is 243

3) find the 10th term

$$\text{Given } a = r, t_5 = 243$$

$$ar^4 = 243 \quad \text{--- (1)}$$

$$r \cdot r^4 = 243$$

$$r^5 = 3^5$$

$$r = 3 \cdot r = 3$$

$$\begin{aligned} t_{10} &= a \cdot r^9 \\ &= a \cdot 3^9 \\ &= 3^{10} \end{aligned}$$

22) The sum of the first 6 terms is same as 9 times the sum of the last 3 terms find r .

$$S_6 = 9S_3$$

$$\frac{a(r^6 - 1)}{(r - 1)} = 9 \cdot \frac{a(r^3 - 1)}{r - 1}$$

$$(r^6 - 1) = 9(r^3 - 1)$$

$$\text{let } x = r^3$$

$$x^2 - 1 = 9(x - 1)$$

$$\text{w.k.p } a^x - b^x = (a - b)(a + b)$$

$$\Rightarrow (x - 1)(x + 1) = 9(x - 1)$$

$$\Rightarrow x - 1 = 9 - 1 = 8$$

$$\Rightarrow r^3 = 2^3$$

$$r = 2$$

23) Sum of the 3 numbers of Ap is 35 and product is 1000

let 3 numbers of Ap is $\frac{a}{r}, a, ar$

$$\text{Product} = 1000$$

$$\Rightarrow \frac{a}{r} \times a \times ar = 1000$$

$$\Rightarrow a^3 = 10^3$$

$$a = 10$$

$$\text{Given sum} = 35$$

$$\frac{a}{r} + a + ar = 35$$

$$\frac{a + ar + ar^2}{r} = 35$$

$$\Rightarrow a + ar + ar^2 = 35r$$

$$\Rightarrow a(1+r+r^2) = 35r$$

$$\Rightarrow \frac{10}{2}(1+r+r^2) = \frac{35}{2}r$$

$$\Rightarrow 2 + 2r + 2r^2 = 7r$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow 2r^2 - 4r - r + 2 = 0$$

$$\Rightarrow 2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2 \text{ or } \frac{1}{2}$$

Case 1 : $a=10, r=2$

$$\frac{a}{r} = \frac{10}{2}$$

$$= 5$$

$$a = 10$$

$$ar = 10 \times 2 = 20$$

Case 2 : $a=10, r=\frac{1}{2}$

$$\frac{a}{r} = \frac{10}{\frac{1}{2}} = 20$$

$$a = 10$$

$$ar = 10 \cdot \frac{1}{2} = 5$$

24) Sum of the GP is 38 & their product is 1728

Sol let the 3 numbers of GP is $a/r, a, ar$

$$\text{Product} = 1728$$

$$\Rightarrow \frac{a}{r} \times a \times ar = 1728$$

$$\Rightarrow a^3 = 1728$$

$$a^3 = 12^3$$

$$a = 12$$

$$\text{Given Sum} = 1728 - 38$$

$$\frac{a}{r} + a + ar = 1728 - 38$$

$$\Rightarrow \frac{a + ar + ar^2}{r} = 1728 - 38$$

$$\begin{array}{r} 2 \overline{) 1728} \\ 2 \overline{) 864} \\ 2 \overline{) 432} \\ 2 \overline{) 216} \\ 2 \overline{) 108} \\ 2 \overline{) 54} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ 3 \overline{) 3} \\ 3 \overline{) 1} \end{array}$$

$$26 \times 33 = 1$$

$$\frac{a(1+r+r^2)}{r} = 1728$$

$$a(1+r+r^2) = 1728r$$

$$\frac{12}{6}(1+r+r^2) = 1728r$$

$$1+r+r^2 = 144r$$

$$6+6r+6r^2-144r=0$$

$$6r^2-138r+6=0$$

$$\Rightarrow 6r^2-9r-4r+6=0$$

$$\Rightarrow 3r(2r-3)-2(2r-3)=0$$

$$\Rightarrow (2r-3)(3r-2)=0$$

$$\Rightarrow r = 3/2, 2/3$$

Case 1 : $a=12, r=3/2$

$$\frac{a}{r} = \frac{12}{3/2} = 12 \times \frac{2}{3} = 8$$

$$a = 12$$

$$ar = 12 \times \frac{3}{2} = 18$$

Case 2 : $a=12, r=2/3$

$$\frac{a}{r} = \frac{12}{2/3} = 12 \times \frac{3}{2} = 18$$

$$a = 12$$

$$ar = 12 \times 2/3 = 8$$

Qn) Sum of the 3 numbers is 14 & product is 64

Sol: Given Product = 64

Let 3 No's be $a, ar, a/r$

$$\therefore a \times ar \times a/r = 64$$

$$a^3 = 64$$

$$\therefore a = 4$$

Given Sum = 14

$$a + ar + \frac{a}{r} = 14$$

$$a(r^2 + r + 1) = 14r$$

$$\frac{1}{2}(r^2 + r + 1) = 14r$$

$$2r^2 + 2r - 7r + 2 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2) (4r-1)$$

$$\therefore r = 2 \frac{1}{2}$$

Case - 1:- When $r=2, a=4$

$$a = 4$$

$$ar = 2 \times 4 = 8$$

$$\frac{a}{r} = \frac{4}{2} = 2$$

Case - 2:- When $r = \frac{1}{2}, a=4$

$$a = 4$$

$$ar = \frac{4 \times \frac{1}{2}}{\frac{1}{2}} = 4 \times \frac{1}{2} = \frac{2}{\frac{1}{2}} = 2$$

$$\frac{a}{r} = 4 \div \frac{1}{2}$$

$$= \frac{4}{\frac{1}{2}} \times 2$$

$$\therefore \frac{a}{r} = 8$$

✓ 26) Sum of the numbers is 19 & product is 216

Sol:-

let 3 No's be $a, ar, a/r$

$$\therefore \text{Product} = 216$$

$$a \times ar \times \frac{a}{r} = 216$$

$$a^3 = 6^3$$

$$\boxed{\therefore a = 6}$$

$$\therefore \text{Sum} = 19$$

$$a + ar + \frac{a}{r} = 19$$

$$a + ar^2 + a = 19r$$

$$6r + 6r^2 + 6 - 19r = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(2r-3) - 2(2r-3) = 0$$

$$(2r-3)(3r-2) = 0$$

$$(2r-3)(3r-2) = 0$$

$$\therefore r = 2/3, 3/2$$

Case-1:- $a = 6, r = 2/3$

$$a = 6$$

$$ar = 6 \times 2/3 = 4$$

$$a/r = 6 \times 3/2 = 9$$

$$= 6 \times 2/3 = 12/3 = 4$$

Case-2:- $a = 6, r = 3/2$

$$a = 6$$

$$ar = 6 \times 3/2 = 9$$

$$a/r = 6 \times 2/3 = 4$$

Q7) Insert 3 GM b/w $\frac{3}{49}$ & 147

Sol:- Given $t_1 = a = \frac{3}{49}$

$$t_5 = ar^4 = 147$$

$$\therefore r^4 = \frac{147}{a} = 11$$

$$= \frac{147}{\frac{3}{49}} = 147 \times \frac{49}{3}$$

$$= 7^2 \times 7^2 = 7^4 \quad [\because a^m \cdot a^n = a^{m+n}]$$

$$\therefore r = 7$$

$$\therefore t_1 = \frac{3}{49}$$

$$t_2 = t_1 \times 7 = \frac{3}{49} \times 7 = \frac{3}{7}$$

$$t_3 = t_2 \times 7 = \frac{3}{7} \times 7 = 3$$

$$t_4 = t_3 \times 7 = 3 \times 7 = 21$$

$$t_5 = t_4 \times 7 = 21 \times 7 = 147$$

$\therefore$ The Series is $\frac{3}{49}, \frac{3}{7}, 3, 21, 147$

28) Insert 3 GM b/w 3 & 48

Sol:-

Given $t_1 = a = 3$

$$t_5 = 48 \Rightarrow ar^4 = 48$$

$$r^4 = \frac{48}{3} = 16$$

$$r^4 = 2^4 = 16$$

$$\therefore r = 2$$

$$t_1 = 3$$

$$t_2 = t_1 \times 2 = 6$$

$$t_3 = t_2 \times 2 = 12$$

$$t_4 = t_3 \times 2 = 24$$

$$t_5 = t_4 \times 2 = 48$$

$\therefore$ The Series is 3, 6, 12, 24, 48.

✓
29)

Insert

6 GM b/w

$1/27$ & 81

Sol:-

Given $a = 1/27$

$$t_8 = ar^7 = 81$$

$$r^7 = \frac{81}{a}$$

$$r^7 = \frac{81}{1/27}$$

$$= 81 \times 27 = 3^4 \times 3^3$$

$$= 3^7$$

$$\therefore r^7 = 3^7$$

$$\therefore r = 3$$

$$t_1 = 1/27$$

$$t_2 = t_1 \times 3 = 1/9$$

$$T_3 = T_2 \times 3 = \frac{1}{3}$$

$$T_4 = T_3 \times 3 = 1$$

$$T_5 = T_4 \times 3 = 3$$

$$T_6 = T_5 \times 3 = 9$$

$$T_7 = T_6 \times 3 = 27$$

$$T_8 = T_7 \times 3 = 81$$

∴ The Series is $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, 81$.

30) Insert 4 GM b/w 1, 243

Sol. Given $T_1 = a = 1$.

$$T_6 = ar^5 = 243$$

$$(1) (r^5) = (3)^5$$

$$\boxed{\therefore r = 3}$$

$$T_1 = 1$$

$$T_2 = T_1 \times 3 = 3$$

$$T_3 = T_2 \times 3 = 9$$

$$T_4 = T_3 \times 3 = 27$$

$$T_5 = T_4 \times 3 = 81$$

$$T_6 = T_5 \times 3 = 81 \times 3 = 243$$

∴ Series is 1, 3, 9, 27, 81, 243

31) — , 1 — , — , — , — , $\frac{243}{32}$

Sol. Given $T_2 = ar = 1$

$$T_7 = ar^6 = \frac{243}{32}$$

$$ar \cdot r^5 = \frac{243}{32}$$

$$(1) r^5 = 2\left(\frac{3}{2}\right)^5$$

$$\therefore r = 3/2$$

$$ar = 1 \Rightarrow a = 1/r = 1/(3/2) = 2/3 = T_1$$

$$T_2 = 1$$

$$T_3 = 1 \times 3/2 = 3/2$$

$$T_4 = 3/2 \times 3/2 = 9/4$$

$$T_5 = T_4 \times 3/2 = 27/8$$

$$T_6 = T_5 \times 3/2 = 81/16$$

$$T_7 = T_6 \times 3/2 = 243/32$$

$\therefore$ The Series is $2/3, 1, 3/2, 9/4, 27/8, 81/16, 243/32$.

$$32) \quad _, _, 18, _, _, _, 32/9, _$$

Sol:-

$$\text{Given } T_3 = ar^2 = 18$$

$$T_7 = ar^6 = \frac{32}{9}$$

$$r^8 = \frac{32/9}{18}$$

$$r^8 = (2/3)^4$$

$$\therefore r = 2/3$$

$$ar^2 = 18$$

$$a = \frac{18}{\frac{4}{9}} = \frac{18 \times 9}{4} = \frac{81}{2} = T_1$$

$$T_2 = T_1 \times 2/3 = 27$$

$$T_3 = 18$$

$$T_4 = T_3 \times 2/3 = 12$$

$$T_5 = T_4 \times 2/3 = 8$$

$$T_6 = T_5 \times 2/3 = 16/3$$

$$T_7 = 32/9$$

$$T_8 = T_7 \times 2/3 = 64/27$$

$\therefore$ Series is $\frac{81}{2}, 27, 18, 12, 8, \frac{16}{3}, \frac{32}{9}, \frac{64}{27}$

207118
33) The sum of infinity no's of AP is 15 & sum of their squares is 45 find the series

let $a, ar, ar^2, \dots$

$$a = a, r = r$$

$$S_{\infty} = \frac{a}{1-r} = 15 \quad \text{--- (1)}$$

Square of series $a^2, a^2 r^2, a^2 r^4, \dots$

$$a = a^2, r = r^2$$

$$S_{\infty} = \frac{a^2}{1-r^2} = 45 \quad \text{--- (2)}$$

$$\textcircled{1} \div \textcircled{2}$$

$$\Rightarrow \frac{a}{1-r} \div \frac{a^2}{1-r^2} = \frac{15}{45}$$

$$\frac{a}{1-r} \times \frac{1-r^2}{a^2} = 1/3$$

$$\Rightarrow \frac{(1-r)(1+r)}{a(1-r)} = 1/3$$

$$\Rightarrow 3(1+r) = a$$

$$3 + 3r = a$$

Sub a in Eqn (1)

$$\frac{a}{1-r} = 15$$

$$a = 15 - 15r$$

$$3 + 3r = 15 - 15r$$

$$18r = 12$$

$$r = 2/3$$

Sub Eq (1)

$$\frac{a}{1-r} = 15$$

$$\frac{a}{1-2/3} = 15$$

$$\frac{a}{3-2} = 15$$

$$\frac{a}{1/3} = 15$$

$$3a = 15$$

$$a = 5$$

34) The Sum of the infinity is 16 & Sum of the square of the infinity is $\frac{256}{7}$ find a & r

let $a, ar, ar^2, \dots$

$$a = a, r = r$$

$$S_{\infty} = \frac{a}{1-r} = 16 \quad \text{--- (1)}$$

Square of series $a^2, a^2 r^2, a^2 r^4, \dots$

$$a = a^2, r = r^2$$

$$S_{\infty} = \frac{a^2}{1-r^2} = \frac{256}{7} \quad \text{--- (2)}$$

$$\textcircled{1} \div \textcircled{2}$$

$$\frac{a}{1-r} \div \frac{a^2}{1-r^2} = \frac{16}{\frac{256}{7}}$$

$$\frac{a}{1-r} \times \frac{1-r^2}{a^2} = 16 \times \frac{7}{256}$$

$$\frac{(1-r)(1+r)}{a(1-r)} = \frac{7}{16}$$

$$16(1+r) = 7a$$

$$\Rightarrow \frac{16(1+r)}{7} = a$$

Sub a in Equ (1)

$$\frac{a}{1-r} = 16$$

$$\frac{16(1+r)}{7} a = 16 - 16r$$

$$16 + 16r = 7(16 - 16r)$$

$$\Rightarrow 16 + 16r = 112 - 112r$$

$$\Rightarrow 128r = 96$$

$$r = 96/128 = 3/4$$

$$4, 3, \frac{9}{4}$$

$$\frac{4}{1-\frac{3}{4}} = \frac{4}{\frac{4-3}{4}} = \frac{4}{\frac{1}{4}} = 16 \quad \checkmark$$

$$r = \frac{3}{4}$$

$$16(1-\frac{3}{4})$$

$$4 \times (4-\frac{3}{4})$$

$$= 4$$

$$\frac{16}{1-\frac{9}{16}}$$

$$\frac{256}{7}$$

$$2, 4, 8$$

$$\frac{2}{1-2} = 2$$

$$4, 16, 64$$

$$\frac{4}{1-4} = \frac{4}{3}$$

Sub $r=314$ in Equ ①

35) Find the first 2 n terms of the following $8+88+888+ \dots + n \text{ terms}$
w

$$8 [1+11+111+ \dots + n \text{ terms}]$$

Multiply & Divide by 9

$$\Rightarrow \frac{8}{9} [9+99+999+ \dots + n \text{ terms}]$$

$$\rightarrow \frac{8}{9} [(10-1)+(100-1)+(1000-1)+ \dots + n \text{ terms}]$$

$$\Rightarrow \frac{8}{9} [(10)+(10)^2+ \dots + n \text{ terms}] - (1+1+ \dots + n \text{ terms})$$

$$\Rightarrow a=10, r=10$$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1} = \frac{10(10^n-1)}{9}$$

$$\Rightarrow \frac{8}{9} \left[\frac{10(10^n-1)}{9} - n \right]$$

36) Find the first 2 n terms of the following $7+77+ \dots + n \text{ terms}$

$$7 [1+11+111+ \dots + n \text{ terms}]$$

Multiply & Divide by 9

$$\Rightarrow \frac{7}{9} [9+99+ \dots + n \text{ terms}]$$

$$\Rightarrow \frac{7}{9} [(10-1) + (100-1) + \dots + n \text{ terms}]$$

$$\Rightarrow \frac{7}{9} [10 + 10^2 + 10^3 + \dots + n \text{ terms}] - [1 + 1 + \dots + n \text{ terms}]$$

$$\Rightarrow a = 10, r = 10$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{10(10^n - 1)}{10 - 1} = \frac{10(10^n - 1)}{9}$$

$$\Rightarrow \frac{7}{9} \left[\frac{10(10^n - 1)}{9} - n \right]$$

37) Find the first 2 terms of the following $5 + 55 + \dots + n \text{ terms}$

$$5 [1 + 11 + 111 + \dots + n \text{ terms}]$$

Multiply & Divide by 9

$$\Rightarrow \frac{5}{9} [9 + 99 + \dots + n \text{ terms}]$$

$$\Rightarrow \frac{5}{9} [(10-1) + (100-1) + \dots + n \text{ terms}]$$

$$\Rightarrow \frac{5}{9} [10 + 10^2 + 10^3 + \dots + n \text{ terms}] - [1 + 1 + \dots + n \text{ terms}]$$

$$\Rightarrow a = 10, r = 10$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{10(10^n - 1)}{10 - 1} = \frac{10(10^n - 1)}{9}$$

$$\Rightarrow \frac{5}{9} \left[\frac{10(10^n - 1)}{9} - n \right]$$

38) Find the first 2 terms of the following $9 + 99 + 999 + \dots + n \text{ terms}$

$$S_n = \frac{9}{9} \left[\frac{10(10^n - 1)}{9} - n \right]$$

23/7/18

39) Evaluate $0.\overline{45}$

$$0.454545 + \dots \infty$$

$$0.45 + 0.0045 + 0.000045 + \dots \infty$$

$$\frac{45}{100} + \frac{45}{10000} + \frac{45}{1000000} + \dots \infty$$

$$\frac{45}{100} \left[1 + \frac{1}{100} + \frac{1}{10000} + \dots \infty \right]$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1 - 1/100} = \frac{1}{\frac{100-1}{100}} = \frac{100}{99}$$

$$= \frac{45}{100} \left[\frac{100}{99} \right]$$

$$= 5/11$$

40) $0.\overline{23}$

$$0.2323 + \dots \infty$$

$$0.23 + 0.0023 + 0.000023 + \dots \infty$$

$$\frac{23}{100} + \frac{23}{10000} + \frac{23}{1000000} + \dots \infty$$

$$\frac{23}{100} \left[1 + \frac{1}{100} + \frac{1}{10000} + \dots \infty \right]$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1 - 1/100} = \frac{1}{\frac{100-1}{100}} = \frac{100}{99}$$

$$= \frac{23}{100} \left[\frac{100}{99} \right]$$

$$= 23/99$$

41) $0.\overline{456}$

$$0.456456456 + \dots \infty$$

$$0.456 + 0.000456 + 0.000000456 + \dots \infty$$

$$\frac{456}{1000} + \frac{456}{1000000} + \frac{456}{1000000000} + \dots \infty$$

$$\frac{456}{1000} \left[1 + \frac{1}{1000} + \dots \infty \right]$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1 - \frac{1}{1000}} = \frac{1}{\frac{1000-1}{1000}} = \frac{1000}{999}$$

$$\frac{456}{1000} \left[\frac{1000}{999} \right] = \frac{152}{333}$$

42) $0.4\bar{5}$

45) $0.45555\dots \infty$

$$0.45 + 0.05 + 0.005 + \dots \infty$$

$$\frac{4}{10} + \frac{5}{100} + \frac{5}{1000} + \dots \infty$$

$$\frac{4}{10} + \frac{5}{100} \left[1 + \frac{1}{10} + \dots \infty \right]$$

$$a = 1, r = 1/10$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1 - 1/10} = \frac{1}{\frac{10-1}{10}} = \frac{10}{9}$$

$$\frac{4}{10} + \frac{5}{100} \left[\frac{10}{9} \right]$$

$$= \frac{4}{10} + \frac{5}{90} = \frac{36 + 5}{90} = \frac{41}{90}$$

43) $0.2\bar{3}$

46) $0.23333\dots \infty$

$$\Rightarrow 0.2 + 0.03 + 0.003 + \dots \infty$$

$$\Rightarrow \frac{2}{10} + \frac{3}{100} + \frac{3}{1000} + \dots \infty$$

$$\Rightarrow \frac{2}{10} + \frac{3}{100} \left[1 + \frac{1}{10} + \dots \infty \right]$$

$$a = 1, r = 1/10$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1 - 1/10} = \frac{1}{\frac{10-1}{10}} = \frac{10}{9}$$

$$\Rightarrow \frac{2}{10} + \frac{3}{90} = \frac{18+3}{90} = \frac{21}{90}$$

44) $0.2\overline{50}$

$$0.2505050 \dots \infty$$

$$0.2 + 0.050 + 0.00050 + \dots \infty$$

$$\frac{2}{10} + \frac{5}{1000} + \frac{5}{1000000} + \dots \infty$$

$$a = 1, \quad r = 1/100$$

$$S_{\infty} = \frac{1}{1 - 1/100} = \frac{1}{\frac{100-1}{100}} = \frac{100}{99}$$

$$\frac{2}{10} + \frac{5}{100} \left[\frac{100}{99} \right]$$

$$= \frac{198+50}{990} = \frac{248}{990} = \frac{124}{495}$$

* $0.2\overline{34}$
 * $0.\overline{7}$
 * $0.4\overline{56}$

45) $0.6 + 0.66 + 0.666 + \dots$

u7

$$6[0.1 + 0.11 + 0.111 + \dots \text{ } n \text{ terms}]$$

$$\frac{6}{9}[0.9 + 0.99 + 0.999 + \dots \text{ } n \text{ terms}]$$

$$\Rightarrow \frac{6}{9} \left[\frac{99}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \text{ } n \text{ terms} \right]$$

$$\Rightarrow \frac{6}{9} \left[\frac{10-1}{10} + \frac{100-1}{100} + \frac{1000-1}{1000} + \dots \text{ } n \text{ terms} \right]$$

$$\Rightarrow \frac{6}{9} \left[1 - \frac{1}{10} + 1 - \frac{1}{100} + \dots \text{ } n \text{ terms} \right]$$

$$\Rightarrow \frac{6}{9} \left[1+1+\dots \text{ } n \text{ terms} - \left(\frac{1}{10} + \frac{1}{100} + \dots \text{ } n \text{ terms} \right) \right]$$

$$\Rightarrow \frac{6}{9} \left[n - \left(\frac{1}{10} + \frac{1}{100} + \dots \text{ } n \text{ terms} \right) \right]$$

$$a = \frac{1}{10}, \quad r = \frac{1}{10} < 1$$

$$= \frac{1}{10} \left(\frac{10^n - 1}{10} \right)$$

$$= \frac{6}{9} \left[n - \frac{(10^n - 1)}{9 \times 10^n} \right]$$

$$= \frac{2}{3} \left[n - \frac{(10^n - 1)}{9 \times 10^n} \right]$$

21/7/18
46) A person is entitled to receive ^{annual payment.} Every year for which it is less by $\frac{1}{10}$ th of what it was in the first year. If the first payment is Rs. 100 how much will he receive till the end of his life.

Sol) Given first payment = $P_1 = 100$

Second Payment is less than the first by 100

$$\therefore t_2 = t_1 - \frac{1}{10} \text{th of } t_1$$

$$t_2 = t_1 - \frac{1}{10} t_1$$

$$\Rightarrow 100 - \frac{1}{10}(100) = 100 - \frac{100}{10} = 90$$

$$r = \frac{t_2}{t_1} = \frac{90}{100} = \frac{9}{10}$$

$\therefore$ total amt which he will receive till the end of his life

$$S_{\infty} = \frac{a}{1-r} = \frac{100}{1-\frac{9}{10}} = \frac{100}{\frac{10-9}{10}} = 100 \times \frac{10}{1} = 1000$$

47) A sum of Rs. 312 is divided among 4 Persons a, b, c, d in such a way that they form GP if a & d Put together receive 252 find the amt received by Everyone.

$$\text{Total amt} = 312$$

$$S_4 = A + B + C + D = 312$$

$$\therefore S_4 = \frac{a(r^4 - 1)}{r - 1} = 312$$

Given, $A + D = 252$

$$\therefore B + C = 312 - 252 = 60$$

$$A + D = 252$$

$$t_1 + t_4 = 252$$

$$a + ar^3 = 252$$

$$a(1+r^3) = 252 \quad \text{①}$$

$$B + C = 60$$

$$t_2 + t_3 = 60$$

$$ar + ar^2 = 60$$

$$ar(1+r) = 60 \quad \text{②}$$

KLMT, $(a^3 + b^3) = (a+b)(a^2 + ab + b^2)$

$$\frac{\text{①}}{\text{②}} = \frac{252}{60} = \frac{a(1+r^3)}{ar(1+r)}$$

$$\frac{(1+r)(1-r+r^2)}{r(1+r)} = \frac{21}{5}$$

$$5[(1+r)(1-r+r^2)] = 21[r(1+r)] \Rightarrow 5 - 5r + 5r^2 = 21r$$

$$5[1 - r + r^2 + r - r^2 + r^3] = 21[r + r^2] \Rightarrow 5r^3 - 26r + 5 = 0$$

$$5 - 5r + 5r^2 + 5r - 5r^2 + 5r^3 = 21r + 21r^2 \Rightarrow 5r^3 - 26r - r + 5 = 0$$

$$\Rightarrow 5r(r-5) - 1(r-5) = 0$$

$$\Rightarrow 5 - 5r + 5r^2 = 21r$$

$$r = 5, \frac{1}{5}$$

Case 1: $r = 5$

$$\text{②} \Rightarrow a(5)(1+5) = 60$$

$$\Rightarrow 5a(6) = 60$$

$$\Rightarrow 30a = 60 \Rightarrow a = 2$$

Case 2: $r = \frac{1}{5}$

$$\text{②} \Rightarrow a\left(\frac{1}{5}\right)\left(1 + \frac{1}{5}\right) = 60$$

$$\Rightarrow \frac{a}{5} \left(\frac{6}{5}\right) = 60$$

$$\Rightarrow 6a = 1500$$

$$a = 250$$

$$A = t_1 = a = 2$$

$$B = t_2 = ar = 2 \times 5 = 10$$

$$C = t_3 = ar^2 = 2 \times 5 \times 5 = 50$$

$$D = t_4 = ar^3 = 2 \times 5 \times 5 \times 5 = 250.$$

$$A = t_1 = a = 250$$

$$B = t_2 = ar = 250 \cdot \frac{1}{5} = 50$$

$$C = t_3 = ar^2 = 250 \cdot \frac{1}{25} = 10$$

$$D = t_4 = ar^3 = 250 \cdot \frac{1}{125} = 2.$$

48) A person deposits money for 3 consecutive months in Ap. After 3 months the amount amounted to Rs. 65. If the 2 extremes are multiplied by 3 & mean by 5 they form a Ap. find the amt deposited Each month

let the amt of the 3 months be a, b, c

$$\text{Given, } A+B+C = 65 \quad \text{①}$$

Given, $3A, 5B, 3C$ form an Ap.

$$\Rightarrow 5B - 3A = 3C - 5B$$

$$\Rightarrow 10B + 5B = 3A + 3C$$

$$10B = 3(A+C)$$

$$B = \frac{3(A+C)}{10} \quad \text{②}$$

$$A+B+C = 65$$

$$A+C+B = 65$$

$$\frac{10B}{3} + B = 65$$

$$10B + 3B = 195$$

$$13B = 195$$

$$B = 15$$

Sub B=15 in

$$\therefore A+C = \frac{10B}{3}$$

$$A+C = \frac{150}{3}$$

$$A+C = 50$$

$$B = 15, t_2 = ar = 15, t_1 + t_3 = 50$$

$$a + ar^2 = 50$$

$$\text{Let, } A = \frac{a}{r}$$

$$B = a = 15$$

$$C = ar.$$

$$\therefore A + B + C = 65.$$

$$\frac{a}{r} + a + ar = 65$$

$$a + ar + ar^2 = 65r$$

$$a(1 + r + r^2) = 65r \Rightarrow 15 + 15r + 15r^2 = 65r$$

$$\frac{15}{r^3}(1 + r + r^2) = 65 \Rightarrow 15r^2 - 50r + 15 = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow 3r^2 - 9r - r + 3 = 0$$

$$3r(r-3) - 1(r-3) = 0$$

$$(r-3)(3r-1) = 0$$

$$r = 3 \text{ or } 1/3.$$

Case 1 :

$$a = 15, r = 3$$

$$t_1 = a = 15$$

$$ar = 15 \cdot 3 = 45$$

$$a/r = 15/3 = 5$$

Case 2 :

$$t_1 = a = 15$$

$$ar = 15 \cdot 1/3 = 5$$

$$\frac{a}{r} = 15 \cdot \frac{3}{1} = 45.$$

* (The reciprocals of the terms of AP are said to be in Harmonic Progression) ✓

Harmonic Progression

* The reciprocals of the terms of Ap are said to be in Harmonic Progression

* If a, b, c are in H.p $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.p

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{a+c}{ac} \Rightarrow 2ac = b(a+c)$$

$$\Rightarrow \frac{2ac}{a+c} = b$$

$\therefore b$ is called as Harmonic mean

* Arithmetic mean = $\frac{a+c}{2}$

* Geometric mean = $\sqrt{ac}$

* H.M = $\frac{2ac}{a+c}$

* Am is greater than G.M greater than H.M ($Am > G.M > H.M$)

* $\therefore (G.M)^2 = Am \times H.M$

1) Find the 10th term of Hp $\frac{1}{5}, \frac{1}{8}, \frac{1}{11}$

$$t_1 = a = 5, d = t_2 - t_1 = \frac{1}{8} - \frac{1}{5} = -\frac{3}{40}$$

$$t_n = a + (n-1)d$$

$$= 5 + (n-1)\left(-\frac{3}{40}\right)$$

$$= 5 - \frac{3n-3}{40} = \frac{200-3n+3}{40} = \frac{203-3n}{40}$$

$$t_{10} = \frac{203-3(10)}{40} = \frac{203-30}{40} = \frac{173}{40}$$

$\therefore$ 10th of Hp = $\frac{1}{32}$

2) find t_7 & S_7 of $1/4, 1/7, 1/10, \dots$ which are in Hp

$4, 7, 10, \dots$ are in AP

$$t_1 = 4, t_2 = 7, d = t_2 - t_1 = 7 - 4 = 3$$

$$t_n = a + (n-1)d = 4 + (n-1)3 = 4 + 3n - 3 = 3n + 1$$

$$t_7 = a + 6d = 4 + 6(3) = 22$$

$$\begin{aligned} S_7 &= \frac{n}{2} [2a + (n-1)d] = \frac{7}{2} [8 + (7-1)3] \\ &= \frac{7}{2} [8 + 18] \\ &= \frac{7}{2} [26] = 91 \end{aligned}$$

3) Find t_{10} & S_{10} of $-1/2, 1, 1/4, \dots$ which are in Hp

$$t_1 = a = -2, d = 1 + \frac{2}{4} = 3/2$$

$$t_n = a + (n-1)d = -2 + (n-1)3 = -2 + 3n - 3 = 3n - 5$$

$$t_{10} = 3(10) - 5 = 30 - 5 = 25$$

$$\begin{aligned} S_{10} &= 2(-2) + (10-1)3 \\ &= -4 + 27 \\ &= 23 \end{aligned}$$

* 4) If $t_7 = 2/5$ & $t_{10} = 2/7$ which are in Hp find t_n, t_{25}, S_{25}

$t_7 = 2/5$ & $t_{10} = 2/7$ are in Hp

$\Rightarrow t_7 = 5/2$ & $t_{10} = 7/2$ are in AP

$$t_7 = a + 6d = 5/2 \Rightarrow a = 5/2 - 6d \text{ --- (1)}$$

$$t_{10} = a + 9d = 7\frac{1}{2} = a = 7\frac{1}{2} - 9d \quad \text{--- (2)}$$

$$7\frac{1}{2} - 6d = 7\frac{1}{2} - 9d$$

$$3d = 2\frac{1}{2}$$

$$d = 1\frac{1}{3}$$

$$\begin{aligned} \textcircled{1} \rightarrow a &= 7\frac{1}{2} - 6(1\frac{1}{3}) \\ &= 7\frac{1}{2} - 2 = \frac{5-4}{2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= \frac{1}{2} + (n-1)1\frac{1}{3} \\ &= \frac{1}{2} + \left(\frac{1}{3}n - \frac{1}{3}\right) = \frac{1}{6}n - \frac{1}{6} = \frac{2n-1}{6} \\ &= \frac{3 + 2(n-1)}{6} = \frac{3+2n-2}{6} = \frac{2n+1}{6} \end{aligned}$$

$$\begin{aligned} a &= \\ t_4 &= a + 3d \\ 6a - a &= 3d \\ \frac{5a}{3} &= d \end{aligned}$$

$$t_{25} = \frac{2(25)+1}{6} = \frac{17}{6} = 2\frac{5}{6}$$

$$S_{25} = \frac{n}{2} (t_1 + t_{25}) = \frac{25}{2} \left[\frac{1}{2} + \frac{17}{6} \right] = \frac{25}{2} \left[\frac{18}{6} \right] = \frac{225}{2}$$

25/3/18

5) The 4th term is $1\frac{1}{12}$ & 13th term is $1\frac{1}{4}$ in HP find t_n, S_n, S_{40} of

the series

$$\text{Given } t_4 = 12$$

$$t_4 = a + 3d = 12$$

$$3d = 12 - a$$

$$a = 12 - 3d$$

$$t_{13} = 42$$

$$t_{13} = a + 12d = 42$$

$$a = 42 - 12d$$

$$12 - 3d = 42 - 12d$$

$$-30 = -9d$$

$$d = 30/9 = 10/3$$

$$\Rightarrow a = 12 - 3(10/3)$$

$$= 2$$

$$\therefore \text{In HP } a = 1/2 \\ d = 3/10$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 2 + (n-1)\frac{10}{3} = 2 + \frac{10}{3}n - \frac{10}{3} = \frac{6+10n-10}{3} \end{aligned}$$

$$42 = a + 12d, \frac{5a}{3}$$

$$42 = 212$$

$$a = 2$$

$$t_n = \frac{10n-4}{3} \quad \therefore \text{HP} = \frac{3}{10n-4}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} \left[2(2) + (n-1) \frac{10}{3} \right]$$

$$= \frac{n}{2} \left[4 + \frac{10n}{3} - \frac{10}{3} \right]$$

$$= \frac{n}{2} \left[\frac{12+10n-10}{3} \right]$$

$$= \frac{n}{2} \left[\frac{10n+2}{3} \right] = \frac{10n^2+2n}{6} \Rightarrow \therefore \text{HP} = \frac{6}{10n^2+2n}$$

$$S_{40} = \frac{10(40)^2 + 2(40)}{6} = \frac{10(1600) + 80}{6} = \frac{16080}{6} = 2680$$

$$\therefore \text{H.P} = 1/2680$$

Series : $t_1 = a = 2$

$$t_2 = a + d = 2 + \frac{10}{3} = 2\frac{16}{3}$$

$$t_3 = a + 2d = 2 + 2\left(\frac{10}{3}\right) = 2\frac{26}{3}$$

$$t_4 = a + 3d = 2 + 3\left(\frac{10}{3}\right) = 12$$

$$t_5 = a + 4d = 2 + 4\left(\frac{10}{3}\right) = 2\frac{46}{3}$$

$$t_6 = a + 5d = 2 + 5\left(\frac{10}{3}\right) = 2\frac{56}{3}$$

$$t_7 = a + 6d = 2 + 6\left(\frac{10}{3}\right) = 2\frac{66}{3}$$

$$\therefore \text{series in HP} = \frac{1}{2}, \frac{3}{16}, \frac{3}{26}, \frac{1}{12}, \dots$$

6) The 3rd term & 7th term of HP are $\frac{1}{7}$ & $\frac{1}{15}$ respectively find 2nd term & Sum of 21 terms

7) Insert 4 H.M's b/w $\frac{1}{22}$ & $\frac{1}{2}$

$$\frac{1}{22}, \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \frac{1}{2} \quad \text{in A.P.}$$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6$

$$a = \frac{1}{22}, \quad t_6 = a + 5d = \frac{1}{2}$$

$$\Rightarrow \frac{1}{22} + 5d = \frac{1}{2}$$

$$5d = \frac{1}{2} - \frac{1}{22}$$

$$5d = \frac{11-1}{22} = \frac{10}{22}$$

$$d = \frac{10}{22} \times \frac{1}{5} = \frac{2}{22}$$

$$t_1 = \frac{1}{22}$$

$$t_2 = t_1 + d = \frac{1}{22} + \frac{2}{22} = \frac{3}{22}$$

$$t_3 = t_2 + d = \frac{3}{22} + \frac{2}{22} = \frac{5}{22}$$

$$t_4 = t_3 + d = \frac{5}{22} + \frac{2}{22} = \frac{7}{22}$$

$$t_5 = t_4 + d = \frac{7}{22} + \frac{2}{22} = \frac{9}{22}$$

$$t_6 = t_5 + d = \frac{9}{22} + \frac{2}{22} = \frac{11}{22} = \frac{1}{2}$$

$$\therefore \text{H.P. is } \frac{22}{3}, \frac{22}{5}, \frac{22}{7}, \frac{22}{9}, 2$$

8) Insert 7 H.M's b/w 2 & $\frac{1}{2}$

$$\frac{1}{2} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad 2 \rightarrow \text{A.P.}$$

$$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9$$

$$a = \frac{1}{2}, t_9 = 2$$

$$a + 8d = 2$$

$$8d = 2 - \frac{1}{2}$$

$$8d = \frac{3}{2}$$

$$d = \frac{3}{2} \times \frac{1}{8} = \frac{3}{16}$$

$$t_1 = a = \frac{1}{2}$$

$$t_2 = t_1 + d = \frac{1}{2} + \frac{3}{16} = \frac{8+3}{16} = \frac{11}{16}$$

$$t_3 = t_2 + d = \frac{11}{16} + \frac{3}{16} = \frac{11+3}{16} = \frac{14}{16}$$

$$t_4 = t_3 + d = \frac{14}{16} + \frac{3}{16} = \frac{14+3}{16} = \frac{17}{16}$$

$$t_5 = t_4 + d = \frac{17}{16} + \frac{3}{16} = \frac{20}{16}$$

$$t_6 = t_5 + d = \frac{20}{16} + \frac{3}{16} = \frac{23}{16}$$

$$t_7 = t_6 + d = \frac{23}{16} + \frac{3}{16} = \frac{26}{16}$$

$$t_8 = t_7 + d = \frac{26}{16} + \frac{3}{16} = \frac{29}{16}$$

$$t_9 = t_8 + d = \frac{29}{16} + \frac{3}{16} = \frac{32}{16}$$

$$\therefore \text{H.M series are } \frac{16}{11}, \frac{16}{14}, \frac{16}{17}, \frac{16}{20}, \frac{16}{23}, \frac{16}{26}, \frac{16}{29}, \frac{16}{32}$$

2) Insert 4 Hm's b/w $\frac{2}{3}$ & $\frac{6}{19}$

$$\frac{a}{3} \text{ --- --- --- } \frac{6}{19} \rightarrow \text{HP } \frac{3}{2} \text{ --- --- --- } \frac{19}{6}$$

$$a = \frac{3}{2} \quad t_6 = \frac{19}{6}$$

$$t_6 = a + 5d = \frac{19}{6}$$

$$\Rightarrow 5d = \frac{19}{6} - \frac{3}{2} = \frac{19-9}{6} = \frac{10}{6} \times \frac{1}{5} = \frac{10}{30} = \frac{1}{3}$$

$$t_1 = a = \frac{3}{2}$$

$$t_2 = a + d = \frac{3}{2} + \frac{10}{6} = \frac{9+10}{6} = \frac{19}{6}$$

$$t_3 = a + 2d = \frac{3}{2} + \frac{20}{6} = \frac{9+20}{6} = \frac{29}{6}$$

$$t_2 = a + d = \frac{3}{2} + \frac{1}{3} = \frac{9+2}{6} = \frac{11}{6}$$

$$t_3 = t_2 + d = \frac{11}{6} + \frac{1}{3} = \frac{11+2}{6} = \frac{13}{6}$$

$$t_4 = t_3 + d = \frac{13}{6} + \frac{1}{3} = \frac{13+2}{6} = \frac{15}{6}$$

$$t_5 = t_4 + d = \frac{15}{6} + \frac{1}{3} = \frac{15+2}{6} = \frac{17}{6}$$

$$t_6 = t_5 + d = \frac{17}{6} + \frac{1}{3} = \frac{17+2}{6} = \frac{19}{6}$$

$\therefore$ HP $\frac{2}{3}, \frac{16}{11}, \frac{16}{13}, \frac{16}{15}, \frac{16}{17}, \dots$

3) Insert 7 Hm's b/w 1 & $\frac{1}{3}$

$$1 \text{ --- --- --- } \frac{1}{3} \rightarrow \text{HP}$$

$$1 \text{ --- --- --- } \frac{1}{3} \rightarrow \text{AP}$$

$$a = 1, \quad t_9 = \frac{1}{3}$$

$$a + 8d = \frac{1}{3}$$

$$8d = \frac{1}{3} - 1$$

$$8d = -\frac{2}{3}$$

$$d = -\frac{2}{3} \times \frac{1}{8} \Rightarrow d = -\frac{1}{12} \quad \therefore \text{HP} = -12$$

$$t_1 = a = 1$$

$$t_2 = a + d = 1 + \left(-\frac{1}{12}\right) = \frac{12-1}{12} = \frac{11}{12}$$

$$t_3 = a + 2d = \frac{10}{12}$$

$$t_4 = a + 3d = \frac{9}{12}$$

$$t_5 = a + 4d = \frac{8}{12}$$

$$t_6 = a + 5d = \frac{7}{12}$$

$$t_7 = a + 6d = \frac{6}{12}$$

$$t_8 = a + 7d = \frac{5}{12}$$

$$t_9 = a + 8d = \frac{4}{12} //$$

11) The sum of the 3 numbers in HP is 26 & 3rd number is twice the first number.

Let the nos of HP be a, b, c

Given, $c = 2a$ — (1)

Given $a+b+c = 26 \Rightarrow a+b+2a = 26$ (from Eq 1)

$$\Rightarrow 3a + b = 26$$

$$\Rightarrow b = 26 - 3a \text{ — (2)}$$

W.K.T, HM of a, b, c is $b = \frac{2ac}{a+c}$

$$= \frac{2a(2a)}{a+2a} = \frac{4a^2}{3a} = \frac{4a}{3} = b \text{ — (3)}$$

Substituting in Eq 2

$$\frac{4a}{3} = 26 - 3a$$

$$\Rightarrow \frac{4a}{3} + 3a = 26$$

$$\Rightarrow 4a + 9a = 78$$

$$\Rightarrow 13a = 78 \Rightarrow a = 6$$

Substitute in Eq 1 $\Rightarrow c = 2(6)$
 $= 12$

$$\frac{2 \times 4}{2} = 2$$

$$\sqrt{\frac{2 \times 4}{3+5}} = 2\sqrt{2}$$

$$\frac{3+5}{2 \times 5} = 4$$

12) AM of 2 No's is 17 & GM is 4 find HM

Let the 2 nos be a, b

W.K.T, AM of any 2 nos is $A.M = \frac{a+b}{2}$

$$\Rightarrow 17 = \frac{a+b}{2} \Rightarrow 34 = a+b$$

W.K.T, GM of any 2 nos is $G.M = \sqrt{ab}$

$$4 = \sqrt{ab}$$

SoBs

$$ab = 16$$

$$H.M = \frac{2ab}{a+b} = \frac{2(16)}{34} = \frac{16}{17}$$

* Properties of Arithmetic Progression:

- If a fixed real number is added to each of the terms of AP the result is also A.P.
- If every term of AP is multiplied or divided by a fixed number then the resulting series is also A.P.
- Reciprocal of AP is HP.

* Properties of Geometric Progression

- If every term of GP is multiplied or divided by a fixed number then the resulting is also GP.
- If every term of the GP is raised to same power then the resulting is also GP.
- Reciprocal of GP is also GP.

* Q) If a, b, c are in A.P then prove that $b+c, c+a, a+b$ are in A.P

Ans Given, a, b, c are in A.P

$$t_2 - t_1 = t_3 - t_2$$

$$\Rightarrow b - a = c - b$$

$$\Rightarrow b - a + c - c = c - b + a - a$$

$$\Rightarrow b + c - a - c = c + a - b - a$$

$$\Rightarrow (b+c) - (a+c) = (a+c) - (b+a)$$

$$\Rightarrow \underset{t_2}{(b+c+a)} - \underset{t_1}{(b+c)} = \underset{t_3}{(a+b)} - \underset{t_2}{((c+a))}$$

$$\therefore (b+c)(c+a)(a+b) \Rightarrow AP$$

(or)

Given a, b, c are in A.P

$$t_2 - t_1 = t_3 - t_2$$

$$b - a = c - b \Rightarrow 2b = a + c \Rightarrow a + c / 2 = b \text{ --- ①}$$

To prove $b+c, c+a, a+b$ are in A.P

let $b+c, c+a, a+b$ are in A.P

$$\Rightarrow t_2 - t_1 = t_3 - t_2$$

$$(c+a) - (b+c) = (a+b) - (c+a)$$

$$c+a-b-c = a+b-c-a$$

$$a-b = b-c$$

$$a+c = 2b$$

$$\frac{a+c}{2} = b \quad \text{--- (2)}$$

$\therefore$ hence $(b+c), (c+a), (a+b)$ are in A.P

14) If a^x, b^x, c^x are in A.P & $a+b, b+c, c+a$ are $\neq 0$ then
 wh. Prove that $b+c, c+a, a+b$ are in H.P

Given a^x, b^x, c^x are in A.P
 t_1, t_2, t_3

$$t_2 - t_1 = t_3 - t_2$$

$$\Rightarrow b^x - a^x = c^x - b^x \quad \text{--- (1)}$$

To prove $b+c, c+a, a+b$ are in H.P

let $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P
 t_1, t_2, t_3

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b+c) - (c+a)}{(c+a)(b+c)} = \frac{(c+a) - (a+b)}{(a+b)(c+a)}$$

$$\frac{b+c-c-a}{c^2+ab+ac+bc} = \frac{c+a-a-b}{ac+ba+a^2+bc}$$

$$(b+a)a^r = -(c+b)e^r \quad \frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+b)}$$

$$a^r b + a^3 = c^3 + c^r b \Rightarrow (a+b)(b-a) = (b+c)(c-b)$$

from Equ ① & ② $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P

$\therefore b+c, c+a, a+b$ are in H.P

(5) If a, b, c are in H.P $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P
 Given a, b, c are in H.P

$$\Rightarrow b = \frac{2ac}{2(a+c)} \text{ (HM)}$$

To prove $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P

$$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c} \text{ are in A.P}$$

$t_1 \quad t_2 \quad t_3$

$$\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$$

$$\frac{a(c+a) - (b+c)b}{ab} = \frac{b(a+b) - (c+a)c}{cb}$$

$$\frac{ac + a^r - b^r - cb}{ab} = \frac{ba + b^r - c^r - ac}{cb}$$

$$\frac{ac + a^r - b^r - cb}{a} = \frac{ab + b^r - c^r - ac}{c}$$

$$\Rightarrow \frac{ac - bc + a^r - b^r}{a} = \frac{ab - ac + b^r - c^r}{c}$$

$$\Rightarrow \frac{c(a-b) + (a-b)(a+b)}{a} = \frac{a(b-c) + (b+c)(b-c)}{c}$$

$$\frac{a-b(a+b+c)}{a} = \frac{(b-c)(a+b+c)}{c}$$

$$c(a-b) = (b-c)a$$

$$\Rightarrow ac - bc = ba - ac$$

$$\Rightarrow ac + ac = ab + bc$$

$$\Rightarrow 2ac = ab + bc$$

$$\Rightarrow ac = \frac{(a+c)b}{2} \Rightarrow \frac{2ac}{a+c} = b$$

16) If a, b, c are in A.P then show that $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in H.P

12) If a, b, c are in H.P then p.T $a(b+c), b(c+a), c(a+b)$ are in A.P

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{a-b}{ab} = \frac{b-c}{bc}$$

$$(a-b)c = (b-c)a$$

$$ac - bc = ba - ca$$

$$2ac = ba + bc$$

$$\frac{2ac}{a+c} = b \quad \text{--- (1)}$$

To Prove $a(b+c), b(c+a), c(a+b)$ are in A.P

$$b(c+a) - a(b+c) = c(a+b) - (b)(c+a)$$

$$\Rightarrow b(c+a) + b(c+a) = c(a+b) + a(b+c)$$

$$\Rightarrow 2b(c+a) = c(a+b) + a(b+c)$$

$$\Rightarrow 2bc + 2ba = ac + cb + ab + ac$$

$$\Rightarrow 2bc + 2ba = 2ac + b(c+a)$$

$$\Rightarrow 2bc + 2ab = 2ac + bc + ab$$

$$\Rightarrow bc + ab = 2ac$$

$$\Rightarrow b = \frac{2ac}{a+c} \quad \text{--- (2)}$$

18) If a, b, c are in GP & $a^x = b^y = c^z$ show that xyz are in HP.

Given a, b, c are in GP

$$\Rightarrow b^r = ac \quad \text{--- (1)}$$

To prove Given $a^x b^y c^z$

$$\text{Let } a^x = b^y = c^z = k$$

$$\begin{array}{c|c|c} a^x = k & b^y = k & c^z = k \\ a = k^{1/x} & b = k^{1/y} & c = k^{1/z} \end{array}$$

from Equ (1)

$$b^r = ac$$

$$\Rightarrow (k^{1/y})^r = k^{1/x} (k^{1/z})$$

$$\frac{r}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\frac{r}{y} = \frac{x+z}{zx}$$

$$y = \frac{2xz}{x+z}$$

LIMITS

* The limit of a function is a fixed value to which a function approaches as the variable approaches for a given value.

* The function approaches the fixed constant in such a way that the absolute value may approach any value

$$\begin{aligned} \text{Eg: } \lim_{x \rightarrow 2} x^2 + 5x + 3 \\ \Rightarrow 2^2 + 5(2) + 3 \\ \Rightarrow 4 + 10 + 3 = 17. \end{aligned}$$

* Methods of calculating limits:

1) Method of Substitution: under this method the value of 'x' is directly substituted in the given function to get the limit of the function. This method is the last step of Every method.

$$\begin{aligned} \text{Eg: } \lim_{x \rightarrow 3} x^3 - 4x + 5 \\ \Rightarrow 3^3 - 4(3) + 5 \\ \Rightarrow 27 - 12 + 5 = 20 \end{aligned}$$

2) Factorisation method: when the given question is in terms of fraction and after direct substitution if we are getting either numerator or denominator or both as zero, then the given function has to be split into factors. After simplification substitution has to be done

$$\text{Eg: } \lim_{x \rightarrow 1} \frac{x^2 + 5x + 6}{(x-1)}$$

Consider, $x^2 + 5x + 6$

$$x^2 + 2x + 3x + 6$$

$$\Rightarrow x(x+2) + 3(x+2)$$

$$\Rightarrow (x+2)(x+3)$$

$$x^2 + 6x - x - 6$$

$$x(x+6) - 1(x+6)$$

$$(x+6)(x-1)$$

$$\therefore \lim_{x \rightarrow 1} \frac{(x+6)(x-1)}{(x-1)} \Rightarrow 1+6=7.$$

$$2) \lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{x^2 - 3x + 2} = 3$$

$$\frac{2(2)^2 - 5(2) + 2}{(2)^2 - 3(2) + 2}$$

$$= \frac{8 - 10 + 2}{4 - 6 + 2}$$

$$= \frac{0}{0} = 0$$

Consider, $2x^2 - 5x + 2$

$$2x^2 - 4x - x + 2$$

$$2x(x-2) - 1(x-2)$$

$$\Rightarrow (x-2)(2x-1)$$

$$\Rightarrow x = 2 \text{ or } 1/2$$

$$x^2 - 3x + 2$$

$$x^2 - 2x - x + 2$$

$$x(x-2) - 1(x-2)$$

$$(x-2)(x-1)$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(2x-1)}{(x-2)(x-1)} = \frac{2x-1}{x-1} = \frac{2(2)-1}{2-1}$$

$$= 3$$

$$3) \lim_{x \rightarrow -3} \frac{x^2 - 4}{x + 2} = -5$$

$$\frac{(-3)^2 - 4}{-3 + 2}$$

$$\Rightarrow \frac{9 - 4}{-3 + 2} = \frac{5}{-1}$$

Consider, $x^2 - 4$

$$(x-2)(x+2)$$

$$\lim_{x \rightarrow -3} \frac{(x-2)(x+2)}{x+2}$$

$$\Rightarrow -3 - 2 = -5$$

$$4) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 3}{x + 4} = \frac{2}{5}$$

$$\frac{1 - 2(1) + 3}{1 + 4}$$

$$\Rightarrow \frac{2}{5}$$

consider, $x^2 - 2x + 3$

$$x^2 - 3x + x + 3$$

$$\Rightarrow x(x-3) - 1(x-3)$$

$$\Rightarrow (x-3)(x-1)$$

$$\lim_{x \rightarrow 1} \frac{(x-3)(x-1)}{x+4}$$

$$\Rightarrow \frac{(1-3)(1-1)}{1+4}$$

Wrong
is not required

consider,

$$5) \lim_{x \rightarrow 1} \frac{2x^3 - 3x^2 + 1}{9x^2 + 8x + 7}$$

$$\frac{2(1)^3 - 3(1)^2 + 1}{9(1)^2 + 8(1) + 7}$$

$$\Rightarrow \frac{2 - 3 + 1}{9 + 8 + 7} = 0/24 = 0$$

24/9/18

* When the functions are given in radical signs then the numerators or denominator or both have to be rationalised, factorised, simplified and then substituted

1) Show that $\lim_{x \rightarrow 0} \frac{\sqrt{2+3x} - \sqrt{2-5x}}{4x}$

Rationalise the numerator

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{2+3x} - \sqrt{2-5x}}{4x} \times \frac{\sqrt{2+3x} + \sqrt{2-5x}}{\sqrt{2+3x} + \sqrt{2-5x}}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(\sqrt{2+3x})^2 - (\sqrt{2-5x})^2}{4x [\sqrt{2+3x} + \sqrt{2-5x}]}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2+3x-2+5x}{4x [\sqrt{2+3x} + \sqrt{2-5x}]}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{8x}{4x [\sqrt{2+3x} + \sqrt{2-5x}]} \Rightarrow \frac{2}{\sqrt{2+3(0)} + \sqrt{2-5(0)}} \Rightarrow \frac{2}{\sqrt{2} + \sqrt{2}} \Rightarrow \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

2) $\lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{x-4} = -1/6$

Rationalise the numerator

$$\Rightarrow \lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{x-4} \times \frac{3 + \sqrt{5+x}}{3 + \sqrt{5+x}}$$

$$\Rightarrow \lim_{x \rightarrow 4} \frac{(3^2) - (\sqrt{5+x})^2}{(x-4)(3 + \sqrt{5+x})}$$

$$\Rightarrow \lim_{x \rightarrow 4} \frac{9 - 5 - x}{(x-4)(3 + \sqrt{5+x})} = \lim_{x \rightarrow 4} \frac{-x + 4}{(x-4)(3 + \sqrt{5+x})} = \lim_{x \rightarrow 4} \frac{-(x-4)}{(x-4)(3 + \sqrt{5+x})}$$

$$\frac{4'}{4(5+\sqrt{9})} = \frac{1}{5+\sqrt{9}} = -\frac{1}{6}$$

$$3) \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x^2} = \frac{1}{2}$$

$$4) \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} = \frac{1}{2\sqrt{2}}$$

$$5) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{3x+1} - \sqrt{5x-1}} = -4$$

$$3) \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x^2} \times \frac{1 + \sqrt{1-x^2}}{1 + \sqrt{1-x^2}}$$

$$\lim_{x \rightarrow 0} \frac{(1)^2 - (\sqrt{1-x^2})^2}{x^2(1 + \sqrt{1-x^2})}$$

$$\lim_{x \rightarrow 0} \frac{1 - 1 - x^2}{x^2(1 + \sqrt{1-x^2})}$$

$$= \lim_{x \rightarrow 0} \frac{-x^2}{x^2(1 + \sqrt{1-x^2})}$$

$$= \frac{1}{1 + \sqrt{1-0}} = \frac{1}{1+1} = \frac{1}{2}$$

$$4) \lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} = \frac{1}{2\sqrt{2}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} \times \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{x+2})^2 - (\sqrt{2})^2}{x(\sqrt{x+2} + \sqrt{2})}$$

$$\lim_{x \rightarrow 0} \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+2}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{0+2}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$5) \lim_{x \rightarrow 1} \frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}} = -4$$

$$\lim_{x \rightarrow 1} = \frac{x^2-1}{\sqrt{3x+1}-\sqrt{5x-1}} \times \frac{\sqrt{3x+1}+\sqrt{5x-1}}{\sqrt{3x+1}+\sqrt{5x-1}}$$

$$= \frac{(x^2-1)(\sqrt{3x+1}+\sqrt{5x-1})}{(x^2-1)(\sqrt{3x+1}-\sqrt{5x-1})}$$

$$\lim_{x \rightarrow 1} \frac{(x^2-1)(\sqrt{3x+1}+\sqrt{5x-1})}{(\sqrt{3x+1})^2 - (\sqrt{5x-1})^2}$$

$$\lim_{x \rightarrow 1} \frac{(x^2-1)(\sqrt{3x+1}+\sqrt{5x-1})}{3x+1-(5x-1)}$$

$$\lim_{x \rightarrow 1} \frac{(x^2-1)(\sqrt{3x+1}+\sqrt{5x-1})}{3x+1-5x+1}$$

$$\lim_{x \rightarrow 1} \frac{(x+1)(x-1)(\sqrt{3x+1}+\sqrt{5x-1})}{-2x+2} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)(\sqrt{3x+1}+\sqrt{5x-1})}{-2x+2}$$

$$\lim_{x \rightarrow 1} \frac{(x+1)(x-1)(\sqrt{3x+1}+\sqrt{5x-1})}{-2(x-1)} = \frac{(1+1)(\sqrt{3(1)+1}+\sqrt{5(1)-1})}{-2}$$

$$= \frac{2(\sqrt{4}+\sqrt{4})}{-2} = \frac{2+2}{-1} = -4$$

Standard limits

$$① \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

Eg: $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$

$$\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{(x - 2)} = 3 \cdot (2)^{3-1} = 3 \cdot 2^2 = 3 \cdot 4 = 12$$

* $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

$$\lim_{x \rightarrow 2} \frac{x^4 - 2^4}{x - 2} = 4 \cdot (2)^{4-1} = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

* $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 4}$

$$\Rightarrow \lim_{x \rightarrow 2} \frac{\frac{x^4 - 16}{x - 2}}{\frac{x^2 - 4}{x - 2}}$$

$$\Rightarrow \lim_{x \rightarrow 2} \frac{\frac{x^4 - 2^4}{(x - 2)}}{\frac{x^2 - 2^2}{(x - 2)}}$$

$$\Rightarrow \frac{4(2)^{4-1}}{2(2)^{2-1}} = \frac{4 \cdot 2^3}{4 \cdot 1} = 8$$

* $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{\frac{x^3 - 3^3}{(x - 3)}}{\frac{x^3 - 3^3}{(x - 3)}}$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{x^3 - 3^3}{x - 3} = \frac{3 \cdot 3^{3-1}}{1} = 3 \cdot 3^2 = 27$$

$$* \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 9}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 3^4}{x^2 - 3^2}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 3^4}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 3^2}{x - 3}$$

$$\Rightarrow \frac{4 \cdot (3)^{4-1}}{2 \cdot (3)^{2-1}} = \frac{4 \cdot 3^3}{2 \cdot 3^1} = \frac{4 \cdot 27}{2 \cdot 3} = \frac{108}{6} = 18.$$

$$* \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^3 - 27}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 3^4}{x^3 - 3^3}$$

$$\lim_{x \rightarrow 3} \frac{x^4 - 3^4}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{x^3 - 3^3}{x - 3}$$

$$\Rightarrow \frac{4 \cdot 3^{4-1}}{3 \cdot (3)^{3-1}} = \frac{4 \cdot 3^3}{3 \cdot 3^2} = 4$$

$$* \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\text{Eg: } \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$$

Multiply & divide by 3

$$\lim_{3x \rightarrow 3 \times 0} \frac{e^{3x} - 1}{\frac{x \times 3}{3}} = \left[\lim_{3x \rightarrow 0} \frac{e^{3x} - 1}{3x} \right] \times 3$$

$$= 1 \times 3 = 3$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \log_e e$$

Any thing dividend by zero = ∞

Any thing divided by $\infty = 0$

$$\therefore \frac{a}{\infty} = 0$$

$$1) f(x) = x^2 + 3x - 4$$

$$f(x+h) = (x+h)^2 + 3(x+h) - 4$$

$$= x^2 + h^2 + 2xh + 3x + 3h - 4$$

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2 + 3x + 3h) - (x^2 + 3x - 4)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x + 4}{h} \Rightarrow \lim_{h \rightarrow 0} \frac{h(2x + h + 3)}{h} \Rightarrow 2x + 3$$

$$2) f(x) = 2x^2 + 4x + 3$$

$$f(x+h) = 2(x+h)^2 + 4(x+h) + 3$$

$$= 2(x^2 + h^2 + 2xh) + 4x + 4h + 3$$

$$= 2x^2 + 2h^2 + 4x + 4h + 2xh + 3$$

$$\therefore \frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 + 4x + 4h + 3 - (2x^2 + 4x + 3)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 + 4x + 4h + 3 - 2x^2 - 4x - 3}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(4x + 2h + 4)}{h}$$

$$= 4x + 2(0) + 4 = 4x + 4$$

$$\therefore \frac{d}{dx} (2x^2 + 4x + 3) = 4x + 4$$

$$3) f(x) = 3x^2 + 5x + 10$$

$$f(x+h) = 3(x+h)^2 + 5(x+h) + 10$$

$$= 3(x^2 + h^2 + 2xh) + 5x + 5h + 10$$

$$= 3x^2 + 3h^2 + 6xh + 5x + 5h + 10$$

$$\frac{d}{dx} f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 5x + 5h + 10 - 3x^2 - 5x - 10}{h}$$

$$\lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 5h}{h} \Rightarrow \lim_{h \rightarrow 0} \frac{h(6x + 3h + 5)}{h} \Rightarrow 6x + 3(0) + 5 = 6x + 5$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\frac{d}{dx}(uvw) = uvw' + uv'w + u'vw$$

$$1) y = x \log x$$

$$\frac{d}{dx}(y) = x \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} x \right)$$

$$= x \cdot \frac{1}{x} + \log x (1)$$

$$= 1 + \log x$$

$$2) y = x \cdot e^x$$

$$\frac{d}{dx}(y) = x \left(\frac{d}{dx} e^x \right) + e^x \left(\frac{d}{dx} x \right)$$

$$\frac{dy}{dx} = x \cdot e^x + e^x (1)$$

$$= x e^x + e^x \Rightarrow e^x (x+1)$$

$$3) y = a^x + e^x$$

$$\frac{d}{dx}(y) = \frac{d}{dx}(a^x) + \frac{d}{dx} e^x$$

$$= a^x \log a$$

$$3) y = a^x e^x$$

$$\frac{d}{dx}(y) = a^x \left(\frac{d}{dx} e^x \right) + e^x \left(\frac{d}{dx} a^x \right)$$

$$= a^x \cdot e^x + e^x \cdot a^x \log a$$

$$= a^x e^x (1 + \log a)$$

$$4) y = a^x e^x x^2$$

$$\frac{d}{dx}(y) = \left(\frac{d}{dx} e^x \right) x^2 + e^x \frac{d}{dx} x^2$$

$$= e^x \cdot x^2 + e^x \cdot 2x^{2-1}$$

$$= e^x \cdot x^2 + e^x \cdot 2x$$

$$\Rightarrow e^x (x^2 + 2x)$$

$$3) y = \frac{x^2 - 2x + 3}{x-1}$$

Quotient Rule: $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$u = x^2 - 2x + 3 \quad v = x - 1$$

$$\frac{du}{dx} = 2x - 2 \quad \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{(x-1)(2x-2) - (x^2-2x+3)(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - 2x + 2 - x^2 + 2x - 3}{(x-1)^2}$$

$$= \frac{x^2 - 1}{(x-1)^2}$$

$$4) y = \frac{x^2 - 5x + 6}{x^2 + x - 1}$$

$$u = x^2 - 5x + 6 \quad v = x^2 + x - 1$$

$$\frac{du}{dx} = 2x - 5 \quad \frac{dv}{dx} = 2x + 1$$

$$\frac{dy}{dx} = \frac{(x^2 + x - 1)(2x - 5) - (x^2 - 5x + 6)(2x + 1)}{(x^2 + x - 1)^2}$$

$$= \frac{(2x^3 - 5x^2 + 2x^2 - 5x - 2x + 5) - (2x^3 + x^2 - 10x^2 - 5x + 12x + 6)}{(x^2 + x - 1)^2}$$

$$= \frac{-7x^2 + 14x - 1}{(x^2 + x - 1)^2}$$

$$5) y = \frac{3x^2 + 5x}{7x + 4}$$

$$u = 3x^2 + 5x \quad v = 7x + 4$$

$$\frac{du}{dx} = 3(2x) + 5 = 6x + 5 \quad \frac{dv}{dx} = 7(1) = 7$$

$$\frac{dy}{dx} = \frac{(7x+4)(6x+5) - (3x^2+5x)7}{(7x+4)^2}$$

$$\frac{42x^2 + 35x + 24x + 20 - 21x^2 - 35x}{(7x+4)^2} = \frac{21x^2 + 24x + 20}{(7x+4)^2}$$

$$9) y = \frac{(2x+1)(3x+1)}{(4x+1)} = \frac{6x^2+2x+3x+1}{(4x+1)}$$

$$u = 6x^2+5x+1 \quad v = 4x+1$$

$$= 12x+5 \quad = 4(1)$$

$$\frac{dy}{dx} = \frac{(4x+1)(12x+5) - (6x^2+5x+1)(4)}{(4x+1)^2}$$

$$= \frac{48x^2+20x+12x+5 - 24x^2-20x-4}{(4x+1)^2}$$

$$= \frac{24x^2+12x+1}{(4x+1)^2}$$

* Composite function: ~~func~~ function in a function is also called a

Composite function

$$y = \log \sqrt{x^2+2x+6}$$

1) w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} \log(\sqrt{x^2+2x+6}) \cdot \frac{d}{dx} \sqrt{x^2+2x+6} \cdot \frac{d}{dx} (x^2+2x+6)$$

$$= \frac{1}{\sqrt{x^2+2x+6}} \cdot \frac{1}{2\sqrt{x^2+2x+6}} \cdot x(2x+2)$$

$$\Rightarrow \frac{2(x+1)}{2(\sqrt{x^2+2x+6})^2} \Rightarrow \frac{x+1}{x^2+2x+6}$$

2)

D.w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} \log(\sqrt{x^2-2x-1}) \cdot \frac{d}{dx} \sqrt{x^2-2x-1} \cdot \frac{d}{dx} (x^2-2x-1)$$

$$= \frac{1}{\sqrt{x^2-2x-1}} \cdot \frac{1}{2\sqrt{x^2-2x-1}} \cdot 2x-2$$

$$= \frac{2x-2}{2(\sqrt{x^2-2x-1})^2} = \frac{2'(x-1)}{2(\sqrt{x^2-2x-1})^2} = \frac{x-1}{(\sqrt{x^2-2x-1})^2}$$

3) $y = 7^{\sqrt{a^2+x^2}}$

D.w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} 7^{\sqrt{a^2+x^2}} \cdot \frac{d}{dx} \sqrt{a^2+x^2} \cdot \frac{d}{dx} (a^2+x^2)$$

$$\frac{dy}{dx} = 7^{\sqrt{a^2+x^2}} \cdot \log_e 7 \times \frac{1}{2\sqrt{a^2+x^2}} (0+2x)$$

$$7^{\sqrt{a^2+x^2}} \cdot \log_e 7 \cdot \frac{x}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \frac{(7^{\sqrt{a^2+x^2}})(\log_e 7)x}{\sqrt{a^2+x^2}}$$

4) $y = 7^{x^2+2x}$

D.w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx} 7^{x^2+2x} \cdot \frac{d}{dx} x^2+2x$$

$$= 7^{x^2+2x} \log 7^{x^2+2x} \cdot 2x+2$$

$$= 7^{x^2+2x} \log (7^{x^2+2x}) \cdot 2x+2$$

$$\Rightarrow 7^{x^2+2x} \cdot \log 7 \cdot 2x+2$$

$$6) y = e^{\sqrt{4+2x}}$$

D.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} e^{\sqrt{4+2x}} \cdot \frac{d}{dx} \sqrt{4+2x} \cdot \frac{d}{dx} (4+2x)$$

$$= e^{\sqrt{4+2x}} \cdot \frac{1}{2\sqrt{4+2x}} \cdot 2(1)$$

$$= e^{\sqrt{4+2x}} \cdot \frac{2}{2\sqrt{4+2x}} = e^{\frac{\sqrt{4+2x}}{\sqrt{4+2x}}}$$

$$7) y = e^{2x+3}$$

D.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} e^{2x+3} \cdot \frac{d}{dx} (2x+3)$$

$$= e^{2x+3} \cdot 2(1)$$

$$= e^{2x+3} \cdot 2$$

$$8) y = \sqrt{2x+1/x}$$

$$\frac{dy}{dx} = \frac{d}{dx} \sqrt{2x+1/x} \cdot \frac{d}{dx} (2x+1/x)$$

$$= \frac{1}{2\sqrt{2x+1/x}} \cdot 2(1) - 1/x^2$$

$$= \frac{1}{2\sqrt{2x+1/x}} \cdot \frac{2x^2-1}{x^2} = \frac{1}{2\sqrt{2x+1/x}} (2 - 1/x^2)$$

$$= \frac{1}{2\sqrt{2x+1/x}} \cdot \frac{2x^2-1}{x^2}$$

1) $x = at^2$ & $y = 2at$ find $\frac{dy}{dx}$

$$x = at^2$$

d.w.r.t. t

$$\frac{dx}{dt} = a \cdot \frac{d}{dt}(t^2)$$

$$= a \cdot 2t$$

$$= 2at \quad \text{--- (1)}$$

$$y = 2at$$

d.w.r.t. t

$$\frac{dy}{dt} = 2at \Rightarrow \frac{dy}{dt} = 2a \cdot \frac{d}{dt}(t)$$

$$= 2(1)a$$

$$= 2a \quad \text{--- (2)}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

2) $x = \frac{3at}{1+t^3}$ & $y = \frac{3at^2}{1+t^3}$ find $\frac{dy}{dx}$

$$x = \frac{3at}{1+t^3}$$

d.w.r.t. t

$$\frac{dx}{dt} = 3a \cdot \frac{d}{dt} \left(\frac{t}{1+t^3} \right)$$

$$u = 3at \quad v = 1+t^3$$

$$\frac{du}{dt} = 3a(1) \quad \frac{dv}{dt} = 3t^2$$

$$\frac{du}{dt} = 3a$$

$$\frac{dx}{dt} = 3a \cdot \frac{d}{dt} \left(\frac{t}{1+t^3} \right)$$

$$= 3a \cdot \frac{1+t^3(3a) - 3at(3t^2)}{(1+t^3)^2}$$

$$= 3a \cdot \frac{3a + 3at^3 - 9at^3}{(1+t^3)^2}$$

$$\frac{dx}{dt} = 3a \cdot \frac{(3a - 6at^3)}{(1+t^3)^2}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{6at - 3at^4}{(1+t^3)^2}}{\frac{(3a - 6at^3)}{(1+t^3)^2}}$$

$$y = \frac{3at^2}{1+t^3}$$

$$u = 3at^2 \quad v = 1+t^3$$

$$\frac{du}{dt} = 3a(2t) \quad \frac{dv}{dt} = 3t^2$$

$$= 6at$$

$$\frac{dy}{dt} = \frac{(1+t^3)(6at) - (3at^2)(3t^2)}{(1+t^3)^2}$$

$$= \frac{6at + 6at^4 - 9at^4}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{6at - 3at^4}{(1+t^3)^2}$$

$$= \frac{6at - 3at^4}{(1+t^3)^2} \cdot \frac{(1+t^3)^2}{6(3a - 6at^3)}$$

$$= 3at(2at)$$

$$= 3at(2-t^3)$$

$$3a(1-2t^3)$$

$$= \frac{t(2-t^3)}{1-2t^3} = \frac{t(2-t^3)}{-2t^3+1}$$

$$3) \quad x = \frac{1-t^2}{1+t^2} \quad y = \frac{2t}{1+t^2} \quad \text{S.T.} \quad \frac{x}{y} + \frac{dy}{dx} = 0$$

$$x = \frac{1-t^2}{1+t^2}$$

$$y = \frac{2t}{1+t^2}$$

$$u = 1-t^2$$

$$v = 1+t^2$$

$$\frac{du}{dt} = -2t$$

$$\frac{dv}{dt} = 2t$$

$$\frac{dx}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$= \frac{-2t-2t^3 - (2t-2t^3)}{(1+t^2)^2}$$

$$= \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{-4t}{(1+t^2)^2}$$

$$\frac{dy}{dx} = \frac{2-2t^2}{-4t} = \frac{2(1-t^2)}{-4t} = \frac{1-t^2}{-2t}$$

$$u = 2t$$

$$v = 1+t^2$$

$$\frac{du}{dt} = 2$$

$$\frac{dv}{dt} = 2t$$

$$\frac{dy}{dt} = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2 - (4t^2)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$\frac{dy}{dt} = \frac{2-2t^2}{(1+t^2)^2}$$

$$\text{Ans } \frac{x}{y} + \frac{dy}{dx} = 0$$

$$\frac{\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} + \frac{1-t^2}{-2t} = 0$$

$$\frac{1-t^2}{2t} + \frac{1-t^2}{(-2t)} = 0$$

$$\frac{-2t(1-t^2) + 2t(1-t^2)}{(2t)(-2t)} = 0$$

$$\frac{-2t + 2t^3 + 2t - 2t^3}{-4t^2} = 0$$

$$0 = 0$$

(LHS = RHS)

* Implicit function:

$$\frac{d}{dx}(xy) \frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{d}{dx}(uv) = u \left(\frac{d}{dx} v \right) + v \left(\frac{d}{dx} u \right)$$

$$x \left(\frac{dy}{dx} \right) + y \left(\frac{dx}{dx} \right)$$

$$\therefore \frac{d}{dx}(xy) = x \frac{dy}{dx} + y$$

* Solve $x^3 + y^3 - 3axy = 0$

D.w.r.t. 'x'

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) - 3a\left(\frac{d}{dx}(xy)\right) = 0$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} - 3a\left[x \frac{dy}{dx} + y\right] = 0$$

$$3x^2 + 3y^2 \frac{dy}{dx} - 3ax \frac{dy}{dx} - 3ay = 0$$

$$3y^2 \frac{dy}{dx} - 3ax \frac{dy}{dx} = -3x^2 + 3ay$$

$$3 \cdot \frac{dy}{dx} [y^2 - ax] = 3[-x^2 + ay]$$

$$\frac{dy}{dx} = \frac{(-x^2 + ay)}{y^2 - ax}$$

$$\frac{x^3}{3} + 6axy + y^2 + 3y^3 = 1$$

2) $x^2 + y^2 = a^2$

D.w.r.t. 'x'

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} a^2$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$= \frac{-x}{\pm \sqrt{a^2 - x^2}}$$

ST $\frac{x}{y} + \frac{dy}{dx} = 0$

Ans $\frac{x}{\sqrt{a^2 - x^2}} - \frac{x}{\sqrt{a^2 - x^2}} = 0$

3) $y^2 = 4ax$

D.w.r.t. 'x'

$$\frac{d}{dx} y^2 = \frac{d}{dx} 4ax$$

D.w.r.t. 'x'

$$\begin{aligned}
 \frac{d}{dx}(y) &= \frac{d}{dx} \sqrt{4ax} \cdot \frac{d}{dx} 4ax \\
 &= \frac{1}{2\sqrt{4ax}} \cdot 4a \frac{d}{dx}(x) \\
 &= \frac{1}{2\sqrt{4ax}} \cdot 4a(1) \\
 &= \frac{2a}{\sqrt{4ax}} \\
 &= \frac{\sqrt{2a} \cdot 2\sqrt{a} \sqrt{a}}{\sqrt{4} \cdot \sqrt{a} \cdot \sqrt{x}} \\
 &= \frac{2\sqrt{a} \sqrt{a}}{2\sqrt{a} \cdot \sqrt{x}}
 \end{aligned}$$

$$\therefore \frac{dy}{dx} = \sqrt{\frac{a}{x}}$$

$$\begin{aligned}
 4) \quad x^{2/3} + y^{2/3} &= a^{2/3} \\
 y^{2/3} &= a^{2/3} - x^{2/3} \\
 \therefore y &= (a^{2/3} - x^{2/3})^{3/2}
 \end{aligned}$$

D.w.r.t. 'x'

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (a^{2/3} - x^{2/3})^{3/2} \cdot \frac{d}{dx} (a^{2/3} - x^{2/3}) \\
 &= \frac{3}{2} (a^{2/3} - x^{2/3})^{3/2-1} \cdot \frac{d}{dx} (a^{2/3} - x^{2/3}) \\
 &= \frac{3}{2} (a^{2/3} - x^{2/3})^{1/2} \cdot (0 - \frac{2}{3} x^{-1/3}) \\
 &= \frac{3}{2} (a^{2/3} - x^{2/3})^{1/2} \cdot (-\frac{2}{3} x^{-1/3}) \\
 &= (a^{2/3} - x^{2/3})^{1/2} \cdot (-x^{-1/3})
 \end{aligned}$$

* Logarithmic differentiation : When any methods are not possible we apply logs on both sides & then differentiate

1) $y = x^x$
Apply log on both sides

$$\log y = \log x^x$$

$$\log y = x \log x$$

$$\frac{d}{dx} (\log y) = x \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} x \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot (1)$$

$$= x/x + \log x$$

$$\frac{dy}{dx} = y (1 + \log x)$$

$$\frac{dy}{dx} = x^x (1 + \log x)$$

2) $y = x^{1/x}$

Apply log on both sides

$$\log y = \log x^{1/x}$$

$$\log y = \frac{1}{x} \log x$$

$$u = \frac{1}{x}$$

$$\frac{du}{dx} = -\frac{1}{x^2}$$

$$v = \log x$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$\frac{d}{dx} (\log y) = \frac{1}{x} \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} \frac{1}{x} \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} \left(\frac{1}{x} \right) + \log x \left(-\frac{1}{x^2} \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2} \log x$$

$$\frac{dy}{dx} = y \left(\frac{1 - \log x}{x^2} \right)$$

$$\frac{dy}{dx} = x^{1/x} \frac{(1 - \log x)}{x^2}$$

$$3) y = a^x \cdot x^x$$

D.w.r.t. x

$$u = a^x$$

$$\frac{du}{dx} = a^x \log e$$

$$v = x^x$$

$$\frac{dv}{dx} = x \log x$$

$$\frac{d}{dx} \log v = x \left(\frac{d}{dx} \log x \right) + \log x \left(\frac{d}{dx} x \right)$$

$$\frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{x} + \log x \cdot (1)$$

$$\frac{dv}{dx} = v(1 + \log x) = x^x(1 + \log x)$$

$$3) y = a^x x^x$$

Apply 'log' on b.s.

$$\log y = \log(a^x x^x)$$

$$\log y = \log a^x + \log x^x$$

$$\Rightarrow \log y = x \log a + x \log x$$

w.r.t. x

$$\frac{d}{dx} (\log y) \cdot \frac{d}{dx} y = \left[x \frac{d}{dx} \log a + \log a \frac{d}{dx} x \right] + \left[x \frac{d}{dx} \log x + \log x \frac{d}{dx} x \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left[x \cdot 0 + \log a (1) \right] + \left[x \cdot \frac{1}{x} + \log x (1) \right]$$

$$= \log a + \log x$$

6. If $x^n \cdot y^m = (x+y)^{m+n}$. Find $\frac{dy}{dx}$

Sol: Given, $x^n \cdot y^m = (x+y)^{m+n}$

Apply log on both sides

$$\Rightarrow n \log x + m \log y = (m+n) \log(x+y)$$

D. w. r. t. x.

$$\Rightarrow \frac{n}{x} + \frac{m}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y} + \frac{m+n}{x+y} \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{m}{y} - \frac{m+n}{x+y} \right] = \frac{m+n}{x+y} - \frac{n}{x}$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{mx + my - my - ny}{y(x+y)} \right] = \left[\frac{mx + my - nx - ny}{x(x+y)} \right]$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{mx - ny}{y} \right] = \left[\frac{mx - ny}{x} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

^{imp} 7. $x^y = e^{x-y}$. Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Sol: Apply log on both sides.

$$\Rightarrow \log x^y = \log e^{x-y}$$

$$\Rightarrow y \log x = (x-y) \log e.$$

$$\Rightarrow y \log x = (x-y)$$

$$\Rightarrow y \log x + y = x.$$

$$\Rightarrow y(1 + \log x) = x$$

$$\Rightarrow y = \frac{x}{1 + \log x} \left(\frac{u}{v} \right)$$

$$y = \frac{x}{1 + \log x}$$

$$\Rightarrow \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x) \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(1 + \log x)}{(1 + \log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x)(1) - x \cdot \frac{1}{x}}{(1 + \log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + \log x - x}{(1 + \log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

8. $x^y \cdot y^x = c$. Find $\frac{dy}{dx}$ at $x=e$, $y=e$.

Sol: Apply log on both sides

$$\log(x^y \cdot y^x) = \log c$$

$$\Rightarrow \log x^y + \log y^x = \log c$$

$$\Rightarrow y \log x + x \log y = \log c$$

D.w.r.t. x

$$\Rightarrow y \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} y + x \left[\frac{d}{dx} \log y \cdot \frac{d}{dx} y \right] + \log y$$

$$\begin{aligned}
 \frac{d}{dx} x &= \frac{d}{dx} \log e \\
 \Rightarrow \frac{y}{x} + \log x \cdot \frac{dy}{dx} + \frac{x}{y} \cdot \frac{dy}{dx} + \log y &= 0 \\
 \Rightarrow \frac{dy}{dx} \left[\log x + \frac{x}{y} \right] &= -\frac{\log y}{1} - \frac{y}{x} \\
 \Rightarrow \frac{dy}{dx} \left[\frac{y \log x + x}{y} \right] &= - \left[\frac{x \log y + y}{x} \right] \\
 \Rightarrow \frac{dy}{dx} &= \frac{-(x \log y + y)}{x} \times \frac{y}{(y \log x + x)} \\
 \Rightarrow \frac{dy}{dx} \text{ at } x=e, y=e &= \frac{-(e \log e + e)}{e} \times \frac{e}{(e \log e + e)} \\
 &= \frac{-1(1+e)}{(e+1)} \\
 &= -1.
 \end{aligned}$$

* DERIVATIVES OF HIGHER ORDER:-

- If the process of differentiation is carried out further by differentiating the derivative obtained is called differentiation of higher order.

Ex:- Find the 4th order derivative of x^4 .

Given, $y = x^4$

D. w. r. t. x .

$$\frac{dy}{dx} = 4x^3 - 1^{\text{st}} \text{ order.}$$

1. dy

D. w. r. t. x.

$$\Rightarrow \frac{d}{dx} \left(\frac{dy}{dx} \right) = 4 \cdot \frac{d}{dx} x^3$$

$$\Rightarrow \frac{d^2 y}{dx^2} = 4 \cdot 3x^2$$

$$\Rightarrow \frac{d^2 y}{dx^2} = 12x^2 \text{ — 2nd order.}$$

D. w. r. t. x.

$$\Rightarrow \frac{d}{dx} \left(\frac{d^2 y}{dx^2} \right) = 12 \cdot 2x$$

$$\Rightarrow \frac{d^3 y}{dx^3} = 24x \text{ — 3rd order.}$$

D. w. r. t. x.

$$\Rightarrow \frac{d}{dx} \left(\frac{d^3 y}{dx^3} \right) = 24 \cdot \frac{d}{dx} x$$

$$\Rightarrow \frac{d^4 y}{dx^4} = 24 \text{ — 4th order.}$$

1. $y = (\log x)^2$. Prove that $x^2 \cdot \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 2$.

Sol: Given, $y = (\log x)^2$

D. w. r. t. x.

$$\frac{dy}{dx} = \frac{d}{dx} (\log x)^2 \cdot \frac{d}{dx} \log x$$

$$\Rightarrow \frac{dy}{dx} = 2 \log x \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \log x}{x} \left(\frac{u}{v} \right) \text{ — (1)}$$

D.w.s.t.v

$$x \frac{d}{dx} 2 \log x - 2 \log x \cdot \frac{d}{dx} x$$

$$\Rightarrow \frac{\quad}{x^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{x \cdot \frac{2}{x} - 2 \log x \cdot 1}{x^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{2 - 2 \log x}{x^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{2(1 - \log x)}{x^2} \quad - (2)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = x^2 \left[\frac{2(1 - \log x)}{x^2} \right] + x \left[\frac{2 \log x}{x} \right] = -2.$$

$$\Rightarrow 2 - 2 \log x + 2 \log x = 2.$$

$$\therefore \text{L.H.S} = \text{R.H.S.}$$

Hence proved.

2. $y = x^2 + \frac{1}{x^2}$. Prove that $x^2 \frac{d^2 y}{dx^2} + x \cdot \frac{dy}{dx} = 4y$.

Sol: $y = x^2 + x^{-2}$

D.w.s.t.v.

$$\frac{dy}{dx} = \frac{d}{dx} (x^2) + \frac{d}{dx} (x^{-2})$$

$$\frac{dy}{dx} = 2x + (-2)x^{-3}$$

$$= 2x - \frac{2}{x^3} \quad - (1)$$

D.w.s.t.v:

$$\frac{d^2 y}{dx^2} = 2 - 2 \left(\frac{3}{x^4} \right) = 2 - \frac{6}{x^4} \quad - (2)$$

$$L.H.S = x^2 \cdot \frac{d^2 y}{dx^2} + x \frac{dy}{dx}$$

$$\Rightarrow x^2 \left[2 + \frac{6}{x^4} \right] + x \left[2x - \frac{2}{x^3} \right]$$

$$\Rightarrow x^2 \left[\frac{2x^4 + 6}{x^4} \right] + x \left[\frac{2x^4 - 2}{x^3} \right]$$

$$\Rightarrow \frac{1}{x^2} [2x^4 + 6 + 2x^4 - 2]$$

$$\Rightarrow \frac{1}{x^2} [4x^4 + 4]$$

$$\Rightarrow \frac{4}{x^2} [x^4 + 1]$$

$$\Rightarrow 4 \left[x^4 + \frac{1}{x^2} \right] = 4y.$$

$$\therefore L.H.S = R.H.S.$$

Hence proved.

$$L.H.S = x^2 \cdot \frac{d^2 y}{dx^2} + x \frac{dy}{dx}$$

* Economic Applications :

$$\text{Total cost (TC)} = \text{Fixed cost (FC)} + \text{Variable cost (VC)}$$

$$\text{Average cost (AC)} = \frac{\text{Total cost}}{\text{No. of units produced}}$$

$$\text{Marginal cost (MC)} = \frac{d}{dx} (TC)$$

$$\text{Total revenue} = \text{Price} \times \text{no. of units sold.}$$

$$\text{Average revenue} = \frac{\text{Total revenue}}{\text{no. of units produced}}$$

$$\text{Marginal revenue} = \frac{d}{dx} (TR)$$

* The cost function for producing 'x' no. of unit is given by
 $C(x) = \frac{1}{3}x^3 - 5x^2 - 75x + 10$ At what level of output is the
 Marginal cost is minimum what is minimum marginal cost

Given, $C(x) = \frac{1}{3}x^3 - 5x^2 - 75x + 10$

KKT, $\frac{d}{dx} (TC) = MC$

1). w.r.t 'x'

$$\frac{d}{dx} C(x) = \frac{d}{dx} \left(\frac{1}{3}x^3 - 5x^2 - 75x + 10 \right)$$

$$= \frac{1}{3}(3x^2) - 10x - 75$$

$$= \frac{3x^2}{3} - 10x - 75$$

$$MC = x^2 - 10x - 75$$

2). w.r.t 'x'

Step 1:

$$\frac{d}{dx} MC = \frac{d}{dx} (x^2 - 10x - 75)$$

$$= 2x - 10$$

5+15
5x5x3

Step 2: let $\frac{dmc}{dx} = 0$

$\Rightarrow 2x - 10 = 0$

$2x = 10$

$x = 5$

Step 3: $\frac{d}{dx} MC = 2x - 10$
D.w.r.t 'x'

$\frac{d^2 MC}{dx^2} = 2 (+ve)$

$\therefore MC$ is Min at $\bar{x}$ units

$MC = x^2 - 10x + 75$

$= 5^2 - 10(5) + 75$

$= 25 - 50 + 75$

$= 50$

2) The cost function is $\frac{1}{3}x^3 - \frac{9}{2}x^2 + 20x + 100$ At what level of output is MC minimum and what is the minimum MC

Given $c(x) = \frac{1}{3}x^3 - \frac{9}{2}x^2 + 20x + 100$

$\frac{d}{dx} c(x) = \frac{1}{3} \frac{d}{dx} x^3 - \frac{9}{2} \frac{d}{dx} x^2 + 20 \frac{d}{dx} (x) + \frac{d}{dx} 100$

$= \frac{1}{3} (3x^2) - \frac{9}{2} (2x) + 20(1)$

$MC = x^2 - 9x + 20$

Step 1: $MC = x^2 - 9x + 20$

D.w.r.t 'x'

$\frac{d}{dx} (MC) = \frac{d}{dx} x^2 - 9 \frac{d}{dx} x + \frac{d}{dx} (20)$

$= 2x - 9$

step 2: let $\frac{dMC}{dx} = 0$

$$2x - 9 = 0$$

$$x = 9/2$$

step 3: $\frac{d}{dx} MC = 2x - 9$

D.w.r.t.

$$\frac{d^2 MC}{dx^2} = 2 (+ve)$$

$\therefore MC$ is min at $9/2$

step 4. MC at $9/2$

$$MC = x^2 - 9x + 20$$

$$= \left(\frac{9}{2}\right)^2 - 9\left(\frac{9}{2}\right) + 20$$

$$= \frac{81}{4} - \frac{81}{2} + 20$$

$$= \frac{81 - 162 + 80}{4}$$

$$= -1/4$$

3) The total cost function is $x^3 - 10x^2 + 20x$ find at which level of output is the Avg cost min find the minimum Avg cost

$$TC = x^3 - 10x^2 + 20x$$

$$\therefore \text{WKT } AC = \frac{TC}{x}$$

$$= \frac{x^3 - 10x^2 + 20x}{x}$$

$$AC = x^2 - 10x + 20$$

$$\therefore AC = [x^2 - 10x + 20]$$

Step 1

$$AC = x^2 - 10x + 120$$

D.w.r.t 'x'

$$AC = 2x - 10$$

Step 2: let $\frac{dAC}{dx} = 0$

$$\Rightarrow 2x - 10 = 0$$

$$x = 5$$

Step 3: $\frac{d}{dx}(AC) = 2x - 10$

$$\frac{d^2AC}{dx^2} = 2$$

$\therefore AC$ is min at 5 units

Step-4:

AC at 5 units

$$\begin{aligned} AC &= x^2 - 10x + 120 \\ &= 5^2 - 10(5) + 120 \\ &= 25 - 50 + 120 \end{aligned}$$

$$\therefore AC = 95 //$$

* Revenue Analysis: for a certain company the total cost function is $x^3 - 12x^2 + 48x + 11$ and the total revenue function is $83x - 4x^2 - 21$ where x = no. of units. find the output at which the profit is maximum and also find the maximum profit.

Sol WKT, profit = TR - TC

$$= (83x - 4x^2 - 21) - (x^3 - 12x^2 + 48x + 11)$$

$$= 83x - 4x^2 - 21 - x^3 + 12x^2 - 48x - 11$$

$$= -x^3 + 8x^2 + 35x - 32$$

Step 1: $\pi(x) = -x^3 + 8x^2 + 35x - 32$

D.w.r.t. 'x'

$$\pi'(x) = -3x^2 + 16x + 35$$

Step 2: let $\frac{d}{dx} \pi(x) = 0$

$$\Rightarrow -3x^2 + 16x + 35 = 0$$

$$\Rightarrow 3x^2 - 16x - 35 = 0$$

$$\Rightarrow 3x^2 - 21x + 5x - 35 = 0$$

$$\Rightarrow 3x(x-7) + 5(x-7) = 0$$

$$\Rightarrow (x-7)(3x+5) = 0$$

$$x = 7, \quad x = -5/3 \text{ NA}$$

Step 3: $\frac{d}{dx} \pi(x) = -3x^2 + 16x + 35$

D.w.r.t. 'x'

$$\pi''(x) = -6x + 16$$

step 4: $\frac{d^2\pi(x)}{dx^2}$ at $x=7$

$$\Rightarrow -6(7) + 16$$

$$\Rightarrow -42 + 16 = -26 \text{ (-ve)}$$

$\therefore$ profit is max when $x=7$ units

step 5: profit at $x=7$

$$\text{profit} = -x^3 + 8x^2 + 35x - 32$$

$$= (-7)^3 + 8(7)^2 + 35(7) - 32$$

$$= -343 + 392 + 245 - 32 = 262$$

* The cost function of a company is $100 + 0.015x^2$. The company sells at the rate of 3 per unit find maximum profit

Given no. of units sold $x = 2$

$$P = ₹ 3$$

$$\begin{aligned} \text{WKT, } TR &= P \times Q \\ &= 3 \times x \\ &= 3x \end{aligned}$$

$$\text{Given } TC = 100 + 0.015x^2$$

$$\begin{aligned} \text{WKT profit} &= TR - TC \\ &= 3x - (100 + 0.015x^2) \\ &= -0.015x^2 + 3x - 100 \end{aligned}$$

step 1: D.w.r.t. 'x'

$$\frac{d}{dx} \pi(x) = -0.030x + 3$$

step 2: let $\frac{d}{dx} \pi(x) = 0$

$$-0.030x + 3 = 0$$

$$\begin{aligned} -0.030x &= -3 \\ x &= 100 \end{aligned}$$

Step 3 $\frac{d}{dx} \pi(x) = -0.03x + 3$

D.w.r.t. 'x'

$$\frac{d^2}{dx^2} \pi(x) = -0.03 \text{ (-ve)}$$

$\therefore$ profit max at $x = 0$

Step 4 : Profit at $x = 100$

$$= 0.015(100)^2 + 3(100) - 100$$

$$= 150 + 300 - 100 = ₹ 50$$