UNIT I: INTRODUCTION TO OR & LINEAR PROGRAMMING: Introduction-Meaning, Definitions, Origin, Characteristics/Features of OR, Main Phases of OR-Managerial applications, limitations of OR. Linear Programming: Formulation of LPP, assumptions underlying LPP, Graphical method, Exceptional cases-unbounded, inconsistent system of constraints, consistent and no solution, constraints are equations.

INTRODUCTION TO OR &

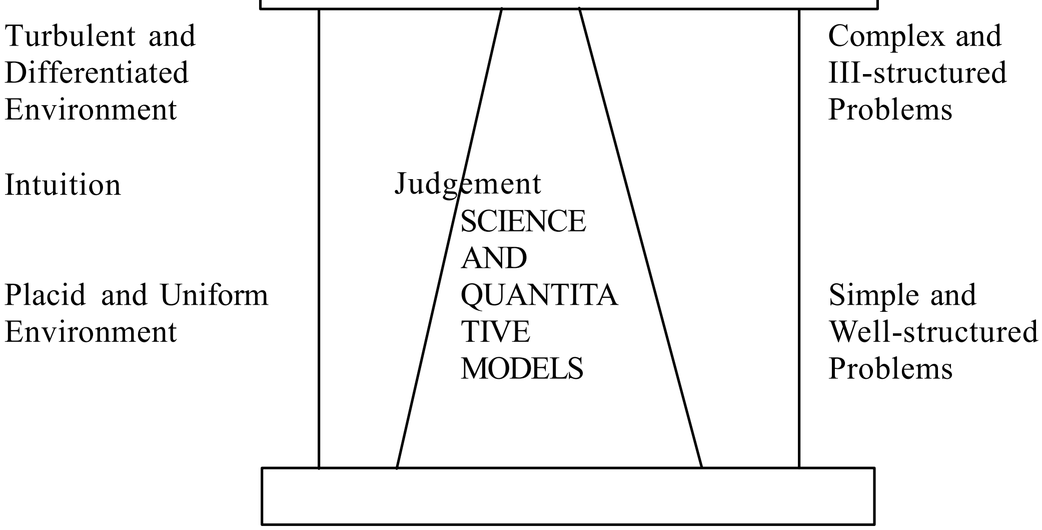
The subject **OPERATIONS RESEARCH** is a branch of mathematics - specially applied mathematics, used to provide a scientific base for management to take timely and effective decisions to their problems. It tries to avoid the dangers from taking decisions merely by guessing or by using thumb rules. Management is the multidimensional and dynamic concept. It is multidimensional, because management problems and their solutions have consequences in several dimensions, such as human, economic social and political fields. As the manager operates his system in an environment, which will never remain static, hence is dynamic in nature. Hence any manager, while making decisions, consider all aspects in addition to economic aspect, so that his solution should be useful in all aspects. The general approach is to **analyse the problem in economic terms and then implement the solution if it does not aggressive or violent to other aspects like human, social and political constraints**.

Management may be considered as the process of integrating the efforts of a purposeful group, or organisation, whose members have at least one common goal. You have studied various schools of management in your management science. Most important among them which uses scientific basis for decision making are:

(*i*) The Decision theory or Decisional Management School and

(*ii*) The Mathematical or Quantitative Measurement School.

The above-mentioned schools of management thought advocate the use of **mathematical methods** or **quantitative methods** for making decisions. Quantitative approach to management problems requires that decision problems be defined, analyzed, and solved in a conscious, rational, logical and systematic and scientific manner - based on data, facts, information and logic, and not on mere guess work or thumb rules. Here we use objectively measured decision criteria. Operations research is the body of knowledge, which uses mathematical techniques to solve management problems and make timely optimal decisions. Operations Research is concerned with helping managers and executives to make better decisions. Today's manager is working in a highly competitive and dynamic environment. In present environment, the manager has to deal with systems with complex interrelationship of various factors among them as well as equally complicated dependence of the criterion of effective performance of the system on these factors, conventional methods of decision-making is found very much inadequate. Though the common sense, experience, and commitment of the manager is essential in making decision, we cannot deny the role-played by scientific methods in making optimal decisions. Operations Research



Intelligence, Information and Data

HISTORY OF OPERATIONS RESEARCH

Operations Research is a **'war baby'**. It is because, the first problem attempted to solve in a systematic way was concerned with *how to set the time fuse bomb to be dropped from an aircraft on to a submarine*. In fact the main origin of Operations Research was during the **Second World War**. At

the time of Second World War, the military management in England invited a team of scientists to study the strategic and tactical problems related to air and land defense of the country. The problem attained importance because at that time the resources available with England was very limited and the objective was to win the war with available meager resources. The resources such as food, medicines, ammunition, manpower etc., were required to manage war and for the use of the population of the country. It was necessary to decide upon the most effective utilization of the available resources to achieve the objective. It was also necessary to utilize the military resources cautiously. Hence, the Generals of military, invited a team of experts in various walks of life such as scientists, doctors, mathematicians, business people, professors, engineers etc., and the problem of resource utilization is given to them to discuss and come out with a feasible solution. These specialists had a brain storming session and came out with a method of solving the problem, which they coined the name **"Linear Programming".** This method worked out well in solving the war problem. As the name indicates, the word **Operations** is used to refer to the problems of military and the word **Research** is use for inventing new method. As this method of solving the problem was invented during the war period, the subject is given the name
'OPERATIONS RESEARCH' and abbreviated as **'O.R.'** After the World War there was a scarcity of industrial material and industrial productivity reached the lowest level. Industrial recession was there and to solve the industrial problem the method **linear programming** was used to get optimal solution. From then on words, lot of work done in the field and today the subject of O.R. have numerous methods to solve different types of problems. After seeing the success of British military, the United States military management started applying the techniques to various activities to solve military, civil and industrial problems. They have given various names to this discipline. Some of them are Operational Analysis, Operations Evaluation, Operations Research, System Analysis, System Evaluation, Systems Research, Quantitative methods, Optimisation Techniques and Management Science etc. But most widely used one is **OPERATIONS RESEARCH.** In industrial world, most important problem for which these techniques used is how to *optimise the profit or how to reduce the costs*. The introduction of Linear Programming and Simplex method of solution developed by American Mathematician George B. Dontzig in 1947 given an opening to go for new techniques and applications through the efforts and co-operation of interested individuals in academic field and industrial field. Today the scenario is totally different. A large number of Operations Research consultants are available to deal with different types of problems. In India also, we have O.R. Society of India (1959) to help in solving various problems. Today the Operations Research techniques are taught at High School levels. To quote some Indian industries, which uses Operations Research for problem solving are: M/S Delhi Cloth Mills, Indian Railways, Indian Airline, Hindustan Lever, Tata Iron and Steel Company, Fertilizers Corporation of India and Defense Organizations. In all the above organizations, Operations Research people act as staff to support line managers in taking decisions.

In one word we can say that Operations Research play a vital role in every organization, especially in decision-making process.

DECISION MAKING AND SOME ASPECTS OF DECISION

Many a time we speak of the word **decision**, as if we know much about decision. But what is decision? What it consists of? What are its characteristics? Let us have brief discussion about the word decision, as much of our time we deal with decision-making process in Operations Research.

A **decision** is the conclusion of a process designed to weigh the relative uses or utilities of a set of alternatives on hand, so that decision maker selects the best alternative which is best to his problem or situation and implement it. **Decision Making** involves all activities and thinking that are necessary to identify the most optimal or preferred choice among the available alternatives. The basic requirements of decision-making are (*i*) A set of goals or objectives, (*ii*) Methods of evaluating alternatives in an objective manner, (*iii*) A system of choice criteria and a method of projecting the repercussions of alternative choices of courses of action. The evaluation of consequences of each course of action is important due to sequential nature of decisions.

The necessity of making decisions arises because of our existence in the world with various needs and ambitions and goals, whose resources are limited and some times scarce. Every one of us competes to use these resources to fulfill our goals. Our needs can be biological, physical, financial, social, ego or higher-level self-actualisation needs. One peculiar characteristics of decision-making is the inherent conflict that desists among various goals relevant to any decision situation (for example, a student thinking of study and get first division and at the same time have youth hood enjoyment without attending classes, OR a man wants to have lot of leisure in his life at the same time earn more etc.). The process of decision-making consists of two phases. The first phase consists of formulation of goals and objectives, enumeration of environmental constraints, identification and evaluation of alternatives. The second stage deals with selection of optimal course of action for a given set of constraints. **In Operations Research, we are concerned with *how to choose optimal strategy under specified set of assumptions*, including all available strategies and their associated payoffs.**

Decisions may be classified in different ways, depending upon the criterion or the purpose of classification. Some of them are shown below:

I Decisions (depending on the purpose)

PURPOSE:	Strategic	Administrative	Operational
	Related to external Environment	Resource utilisation of an orgnisation.	Related to day –to- Repetitive work.

II Decision (Depending on the nature)

Programmed decisions
Meant for repetitive and well-structured problems. Inventory Problems, Product Mix Problems, etc.

Non Programmed decisions
Meant for non-routine, novel, ill-structured problems. Policy matters, Product market mix, plant location Etc.

III Decisions (Depending on the persons involved)

Individual

Managerial

IV. Decisions (Depending on the Sphere of interest)

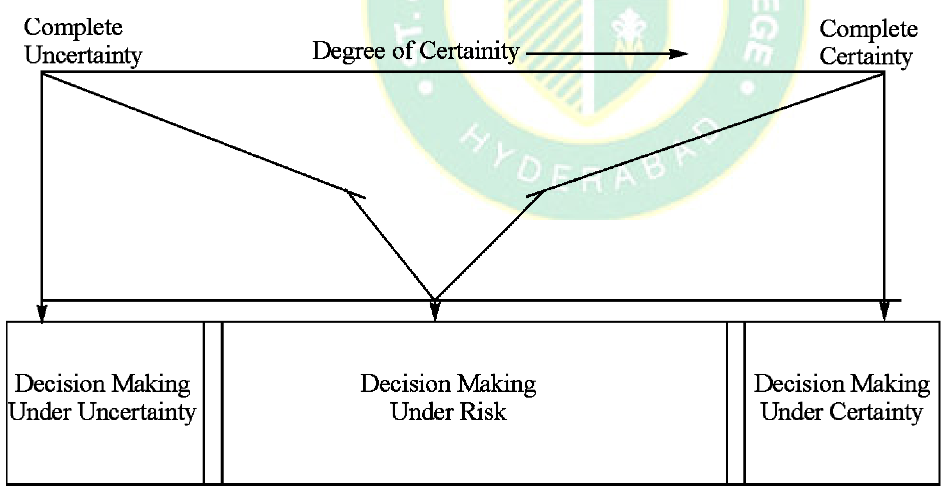
Economic   Social   Business   Political   Social

V. Decisions (depending on the time horizon)

Static
(One decision for entire planning period)

Dynamic
(Decisions are sequential)

Decisions may also be classified depending on the situations such as **degree of certainty**. For example, (*i*) Decision making under certainty (*ii*) Decision making under Uncertainty and (*iii*) Decision making under risk. The first two are two extremes and the third one is falls between these two with certain probability distribution.



Decision based on degree of certainty.

OBJECTIVE OF OPERATIONS RESEARCH

Today's situation in which a manager has to work is very complicated due to complexity in business organizations. Today's business unit have number of departments and each department work for fulfilling the objectives of the organization. While doing so the individual objective of one of the department may be conflicting with the objective of the other department, though both working for achieving the common goal in the interest of the organization. In such situations, it will become a very complicated issue for the general manager to get harmony among the departments and to allocate the available resources of all sorts to the departments to achieve the goal of the organization. At the same time the environment in which the organization is operating is very dynamic in nature and the manager has to take decisions without delay to stand competitive in the market. At the same time a wrong decision or an untimely decision may be very costly. Hence the decision making process has become very complicated at the same time very important in the environment of conflicting interests and competitive strategies. Hence it is desirable for modern manager to use scientific methods with mathematical base while making decisions instead of depending on guesswork and thumb rule methods. Hence the knowledge of Operations Research is an essential tool for a manager who is involved in decision- making process. He must have support of knowledge of mathematics, statistics, economics etc., so that the decision he takes will be an optimal decision for his organisaton. Operation Research provides him this knowledge and helps him to take quick, timely, decisions, which are optimal for the organisaton. Hence the **objective** of operations research is:

"The objective of Operations Research is to provide a scientific basis to the decision maker for solving the problems involving the interaction of various components of an organization by employing a team of scientists from various disciplines, all working together for finding a solution which is in the best interest of the organisaton as a whole. The best solution thus obtained is known as optimal decision".

DEFINITION OF OPERATIONS RESEARCH

Any subject matter when defined to explain what exactly it is, we may find one definition. Always a definition explains what that particular subject matter is. Say for example, if a question is asked what is Boyel's law, we have a single definition to explain the same, irrespective of the language in which it is defined. But if you ask, what Operations research is? The answer depends on individual objective. Say for example a student may say that the Operations research is technique used to obtain first class marks in the examination. If you ask a businessman the same question, he may say that it is the technique used for getting higher profits. Another businessman may say it is the technique used to capture higher market share and so on. Like this each individual may define in his own way depending on his objective. Each and every definition may explain one or another characteristic of Operations Research but none of them explain or give a complete picture of Operations research. But in the academic interest some of the important definitions are discussed below.

(*a*) **Operations Research is the art of winning wars without actually fighting. - Aurther Clarke.**

This definition does not throw any light on the subject matter, but it is oriented towards warfare. It means to say that the directions for fighting are planned and guidance is given from remote area, according to which the war is fought and won. Perhaps you might have read in Mahabharatha or you might have seen some old pictures, where two armies are fighting, for whom the guidance is given by the chief minister and the king with a chessboard in front of them. Accordingly war is fought in the warfront. Actually the chessboard is a model of war field.

(*b*) **Operations Research is the art of giving bad answers to problems where otherwise worse answers are given. - T.L. Satty.**

This definition covers one aspect of decision-making, *i.e.*, choosing the best alternative among the list of available alternatives. It says that if the decisions are made on guesswork, we may face the worse situation. But if the decisions are made on scientific basis, it will help us to make better decisions. Hence this definition deals with one aspect of decision-making and not clearly tells what is operations research.

(*c*) **Operations Research is Research into Operations. - J. Steinhardt.**

This definition does not give anything in clear about the subject of Operations Research and simply says that it is research in to operations. Operations may here be referred as military activities or simply the operations that an executive performs in his organisations while taking decisions. Research in the word means that finding a new approach. That is when an executive is involved in performing his operations for taking decisions he has to go for newer ways so that he can make a better decision for the benefit of his organisation.

(*d*) **Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control. - P.M. Morse and G.E. Kimball.**

This definition suggests that the Operations Research provides scientific methods for an executive to make optimal decisions. But does not give any information about various models or methods. But this suggests that executives can use scientific methods for decision-making.

(*e*) **Operations Research is th study of administrative system pursued in the same scientific manner in which system in Physics, Chemistry and Biology are studied in natural sciences.**

This definition is more elaborate than the above given definitions. It compares the subject Operations Research with that of natural science subjects such as Physics, Chemistry and Biology, where while deciding any thing experiments are conducted and results are verified and then the course of action is decided. It clearly directs that Operations Research can also be considered as applied science and before the course of action is decided, the alternatives available are subjected to scientific analysis and optimal alternative is selected. But the difference between the experiments we conduct in natural sciences and operations research is: in natural sciences the research is rigorous and exact in nature, whereas in operations research, because of involvement of human element and uncertainty the approach will be totally different.

(*f*) **Operations Research is the application of scientific methods, techniques and tools to operation of a system with optimum solution to the problem. - Churchman, Ackoff and Arnoff.**

This definition clearly states that the operations research applies scientific methods to find an optimum solution to the problem of a system. A system may be a production system or information system or any system, which involves men, machine and other resources. We can clearly identify that this definition tackles three important aspects of operations research *i.e.* application of scientific methods, study of a system and optimal solution. This definition too does not give any idea about the characteristics of operations research.

(*g*) **Operations Research is the application of the theories of Probability, Statistics, Queuing, Games, Linear Programming etc., to the problems of War, Government and Industry.**

This definition gives a list of various techniques used in Operations Research by various managers to solve the problems under their control. A manager has to study the problem, formulate the problem, identify the variables and formulate a model and select an appropriate technique to get optimal solution. We can say that operations research is a bunch of mathematical techniques to solve problems of a system.

(*h*) **Operations Research is the use of Scientific Methods to provide criteria or decisions regarding man-machine systems involving repetitive operations.**

This definition talks about man- machine system and use of scientific methods and decision- making. It is more general and comprehensive and exhaustive than other definitions. Wherever a study of system involving man and machine, the person in charge of the system and involved in decision-making will use scientific methods to make optimal decisions.

(*i*) **Operations Research is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with problems that confront the executive, when he tries to achieve a thorough going rationally in dealing with his decision problem. - D.W. Miller and M.K. Starr.**

This definition also explains that operations research uses scientific methods or logical means for getting solutions to the executive problems. It too does not give the characteristics of Operations Research.

(*j*) **Operations Research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, materials and money in industry, business, Government and defense. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcome of alternative decisions, strategies or controls. The purpose is to help management to determine its policy and actions scientifically. - Operations Society of Great Britain.**

The above definition is more elaborate and says that operations research applies scientific methods to deal with the problems of a system where men, material and other resources are involved and the system under study may be industry, defense or business etc, gives this definition. It also say that the manager has to build a scientific model to study the system which must be provided with facility to measure the outcomes of various alternatives under various degrees of risk, which helps the managers to take optimal decisions.

In addition to the above there are hundreds of definitions available to explain what Operations

Research is? But many of them are not satisfactory because of the following reasons.

(*i*) Operations Research is not a well-defined science like Physics, Chemistry etc. All these sciences are having well defined theory about the subject matter, where as operations research do not claim to know or have theories about operations. Moreover, Operations Research is not a scientific research into the control of operations. It is only the application of mathematical models or logical analysis to the problem solving. Hence none of the definitions given above defines operations research precisely.

(*ii*) The objective of operations research says that the decisions are made by brain storming of people from various walks of life. This indicates that operations research approach is inter- disciplinary approach, which is an important character of operations research. This aspect is not included in any of the definitions hence they are not satisfactory.

(*iii*) The above-discussed definitions are given by various people at different times and stages of development of operations research as such they have considered the field in which they are involved hence each definition is concentrating on one or two aspects. No definition is having universal approach.

But salient features of above said definitions are:

* Operations Research uses Scientific Methods for making decisions.
* It is interdisciplinary approach for solving problems and it uses the knowledge and experience of experts in various fields.

* While analyzing the problems all aspects are considered and examined and analyzed scientifically for finding the optimal solution for the problem on hand.
* As operations research has scientific approach, it improves the quality of answers to the problems.
* Operations research provides scientific base for decision-making and provide scientific substitute for judgement and intuition.

CHARACTERISTICS OF OPERATIONS RESEARCH

After considering the objective and definitions of Operations Research, now let us try to understand what are the characteristics of Operations Research.

(*a*) **Operations Research is an interdisciplinary team approach.**

The problems an operations research analyst face is heterogeneous in nature, involving the number of variables and constraints, which are beyond the analytical ability of one person. Hence people from various disciplines are required to understand the operations research problem, who applies their special knowledge acquired through experience to get a better view of cause and effects of the events in the problem and to get a better solution to the problem on hand. This type of team approach will reduce the risk of making wrong decisions.

(*b*) **Operations Research increases the creative ability of the decision maker.**

Operations Research provides manager mathematical tools, techniques and various models to analyse the problem on hand and to evaluate the outcomes of various alternatives and make an optimal choice. This will definitely helps him in making better and quick decisions. A manager, without the knowledge of these techniques has to make decisions by thumb rules or by guess work, which may click some times and many a time put him in trouble. Hence, a manager who uses Operations Research techniques will have a better creative ability than a manager who does not use the techniques.

(*c*) **Operations Research is a systems approach.**

A business or a Government organization or a defense organization may be considered as a system having various sub-systems. The decision made by any sub-system will have its effect on other sub-systems. Say for example, a decision taken by marketing department will have its effect on production department. When dealing with Operations Research problems, one has to consider the entire system, and characteristics or sub- systems, the inter-relationship between sub-systems and then analyse the problem, search for a suitable model and get the solution for the problem. Hence we say Operations Research is a Systems Approach.

SCOPE OF OPERATIONS RESEARCH

The scope aspect of any subject indicates, the limit of application of the subject matter/techniques of the subject to the various fields to solve the variety of the problems. But we have studied in the objective, that the subject Operations Research will give scientific base for the executives to take decisions or to solve the problems of the systems

under their control. The system may be business, industry, government or defense. Not only this, but the definitions discussed also gives different versions. This indicates that the techniques of Operations Research may be used to solve any type of problems. The problems may pertain to an individual, group of individuals, business, agriculture, government or
defense. Hence, we can say that there is no limit for the application of Operations Research methods and techniques; they may be applied to any type of problems. Let us now discuss some of the fields where Operations Research techniques can be applied to understand how the techniques are useful to solve the problems. In general we can state that whenever there is a problem, simple or complicated, we can use operations research techniques to get best solution.

(*i*) In Defense Operations

In fact, the subject Operations research is the baby of World War II. To solve war problems, they have applied team approach, and come out with various models such as resource allocation model, transportation model etc.In any war field two or more parties are involved, each having different resources (manpower, ammunition, etc.), different courses of actions (strategies) for application. Every opponent has to guess the resources with the enemy, and his courses of action and accordingly he has to attack the enemy. For this he needs scientific, logical analysis of the problem to get fruitful results. Here one can apply the techniques like *Linear Programming, Game theory, and inventory models etc.* to win the game. In fact in war filed every situation is a competitive situation. More over each party may have different bases, such as Air force, Navy and Army. The decision taken by one will have its effect on the other. Hence proper co-ordination of the three bases and smooth flow of information is necessary. Here operations research techniques will help the departmental heads to take appropriate decisions.

(*ii*) In Industry

After the II World War, the, Industrial world faced a depression and to solve the various industrial problems, industrialist tried the models, which were successful in solving their problems. Industrialist learnt that the techniques of operations research can conveniently applied to solve industrial problems. Then onwards, various models have been developed to solve industrial problems. Today the managers have on their hand numerous techniques to solve different types of industrial problems. In fact *decision trees, inventory model, Linear Programming model, Transportation model, Sequencing model, Assignment model and replacement models* are helpful to the managers to solve various problems, they face in their day to day work. These models are used to minimize the cost of production, increase the productivity and use the available resources carefully and for healthy industrial growth. An industrial manager, with these various models on his hand and a computer to workout the solutions (today various packages are available to solve different industrial problems) quickly and preciously.

(*iii*) In Planning For Economic Growth

In India we have five year planning for steady economic growth. Every state government has to prepare plans for balanced growth of the state. Various secretaries belonging to different departments has to co-ordinate and plan for steady economic growth. For this all departments can use Operations research techniques for planning purpose. The question like how many engineers, doctors, software people etc. are required in future and what should be their quality to face the then problems etc. can be easily solved.

(*iv*) In Agriculture

The demand for food products is increasing day by day due to population explosion. But the land available for agriculture is limited. We must find newer ways of increasing agriculture yield. So the selection of land area for agriculture and the seed of food grains for sowing must be meticulously done so that the farmer will not get loss at the same time the users will get what they desire at the desired time and desired cost.

(*v*) In Traffic control

Due to population explosion, the increase in the number and verities of vehicles, road density is continuously increasing. Especially in peak hours, it will be a headache to control the traffic. Hence proper timing of traffic signaling is necessary. Depending on the flow of commuters, proper signaling time is to be worked out. This can be easily done by the application of *queuing theory*.

(*vi*) In Hospitals

Many a time we see very lengthy queues of patient near hospitals and few of them get treatment and rest of them have to go without treatment because of time factor. Some times we have problems non-availability of essential drugs, shortage of ambulances, shortage of beds etc. These problems can be conveniently solved by the application of operations research techniques.

The above-discussed problems are few among many problems that can be solved by the application of operation research techniques. This shows that Operations Research has no limit on its scope of application.

PHASES IN SOLVING OPERATIONS RESEARCH PROBLEMS OR STEPS IN SOLVING OPERATIONS RESEARCH PROBLEMS

Any Operations Research analyst has to follow certain sequential steps to solve the problem on hand. The steps he has to follow are discussed below:

First he has to study the situation and collect all information and formulate the statement of the problem. Hence the first step is the *Formulation of the problem*. The figure 1.3 shows the various steps to be followed.

Formulation of the Problem

The Operations Research analyst or team of experts first have to examine the situation and clearly define what exactly happening there and identify the variables and constraints. Similarly identify what is the objective and put them all in the form of statement. The statement must include a) a precise description goals or objectives of the study, b) identification of controllable and uncontrollable variables and c) restrictions of the problem. The team should consult the personals at the spot and collect information, if something is beyond their reach, they have to consult duty engineers available and understand the facts and formulate the problem. Let us consider the following statement:

Statement: A company manufactures **two products *X* and *Y***, by using the **three machines *A*, *B*, and *C*.** Each unit of *X* takes **1 hour** on machine A, **3 hours** on machine B and **10 hours** on machine *C*. Similarly, product *Y* takes **one hour, 8 hours and 7 hours** on Machine *A*, *B*, and *C* respectively. In the coming planning period, **40 hours of machine *A*, 240 hours of machine *B* and 350 hours of machine *C*** is available for production. Each unit of ***X* brings a profit of Rs 5/-** and ***Y* brings Rs. 7 per unit. How much of *X* and *Y* are to be manufactured** by the company **for maximizing the profit**?

The team of specialists prepares this statement after studying the system. As per requirement this must include the variables, constraints, and objective function.

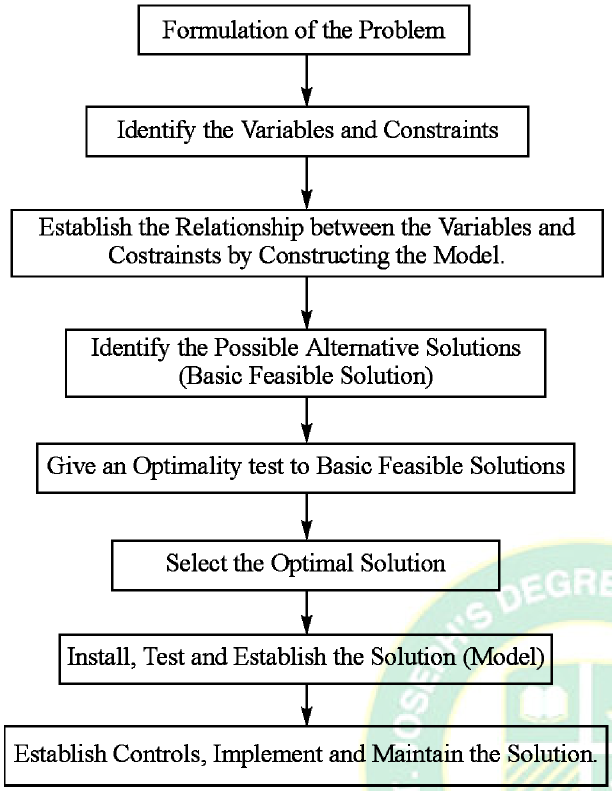


Figure 1.3. Phases of Solving Operations Research Problems.

Variables

The Company is manufacturing two products *X* and *Y*. These are the two variables in the problem. When they are in the problem statement they are written in *capital letters.* Once they are entered in the model small letters (lower case) letters are used (*i.e,*. *x* and *y*). We have to find out how much of *X* and how much of *Y* are to be manufactured. Hence they are variables. In linear programming language, these are known as c**ompeting candidates**. Because they compete to use or consume available resources.

Resources and Constraints

There are three machines *A*, *B*, and C on which the products are manufactured. These are known as resources. The capacity of machines in terms of machine hours available is the available resources. The competing candidates have to use these available resources, which are limited in nature. Now in the above statement, machine *A* has got available 40 hours and machine B has available a capacity of

240 hours and that of machine *C* is 350 hours. The products have to use these machine hours in required proportion. That is one unit of product *X* consumes one hour of machine *A*, 3 hours of machine *B* and 10 hours of machine *C*. Similarly, one unit of *Y*

consumes one hour of machine *B*, 8 hours of machine *B* and 7 hours of machine *C*. These machine hours given are the available resources and they are limited in nature and hence they are **constraints** given in the statement.

Objective of the Problem

To maximise the profit how much of *X* and *Y* are to be manufactured? That is **maximization of the profit or maximization of the returns** is the objective of the problem. For this in the statement it is given that the profit contribution of X is Rs 5/- per unit and that of product Y is Rs. 7/- per unit.

To establish relationship between variables and constraints and build up a model

Let us say that company manufactures x units of *X* and y units of *Y*. Then as one unit of *x* consumes one hour on machine A and one unit of y consumes one hour on machine A, the total consumption by manufacturing *x* units of *X* and y units of *Y* is, 1*x* + 1*y* and this should not exceed available capacity of
40 hours. Hence the **mathematical relationship in the form of mathematical model is 1*x* + 1*y***
40. This is for resource machine *A*. Similarly for machine *B* and machine *C* we can formulate the
mathematical models. They appear as shown below:
3*x* + 8*y* 240 for machine *B* **and 10*x* + 7*y* 350** for machine *C*. Therefore, the mathematical model for these resources are:

1*x* + 1*y* 40
3*x* + 8*y* 240 and
10*x* + 7*y* 350.

Similarly for objective function as the company manufacturing x units of *X* and y units of *Y* and the profit contribution of *X* and Y are Rs.5/- and Rs 7/- per unit of *X* and *Y* respectively, the total profit earned by the company by manufacturing *x* and *y* units is **5*x* + 7*y*.** This we have to maximise. There- fore **objective function is Maximise 5*x* + 7*y*.** At the same time, we have to remember one thing that the company can manufacture any number of units or it may not manufacture a particular product, for example say *x* = 0. But it cannot manufacture negative units of *x* and *y*. Hence one more constraint is to be introduced in the model *i.e.* **a non - negativity constraint.** Hence the mathematical representation of the contents of the statement is as given below:

Maximise Z = 5*x* + 7*y* Subject to a condition (written as s.t.) → **OBJECTIVE FUNCTION.**

1*x* + 1*y* 40
3*x* + 8*y* 240 → **STRUCTURAL CONSTRAINTS.**
10*x* + 7*y* 350 and
Both *x* and *y* are 0 → **NON-NEGATIVITY CONSTRAINT.**

1.8.6. Identify the possible alternative solutions (or known as Basic Feasible Solutions or simply BFS)

There are various methods of getting solutions. These methods will be discussed later. For example we go on giving various values (positive numbers only), and find various values of objective function. All these are various Basic Feasible Solutions. For example

x = 0,1,2,3, etc. and y = 0,1,2,3 etc are all feasible values as far as the given condition is concerned. Once we have feasible solutions on hand go on asking is it maximum? Once we get maximum value, those values of x and y are optimal values. And the value of objective function is **optimal value of the objective function**. These two steps we shall discuss in detail in the next chapter.

Install and Maintain the Solution

Once we get the optimal values of x and y and objective function instructions are given to the concerned personal to manufacture the products as per the optimal solution, and maintain the same until further instructions.

MEANING AND NECESSITY OF OPERATIONS RESEARCH MODELS

Management deals with reality that is at once complex, dynamic, and multifacet. It is neither possible nor desirable, to consider each and every element of reality before deciding the courses of action. It is impossible because of time available to decide the courses of action and the resources, which are limited in nature. More over in many cases, it will be impossible for a manager to conduct experiment in real environment. For example, if an engineer wants to measure the inflow of water in to a reservoir through a canal, he cannot sit on the banks of canal and conduct experiment to measure flow. He constructs a similar model in laboratory and studies the problem and decides the inflow of water. Hence for many practical problems, a model is necessary. We can **define an operations research model as some sort of mathematical or theoretical description of various variables of a system representing some aspects of a problem on some subject of interest or inquiry. The model enables to conduct a number of experiment involving theoretical subjective manipulations to find some optimum solution to the problem on hand.**

Let us take a very simple example. Say you have a small child in your house. You want to explain to it what is an elephant. You can say a story about the elephant saying that it has a trunk, large ears, small eyes etc. The child cannot understand or remember anything. But if you draw small drawing of elephant on a paper and show the trunk, ears, eyes and it will grasp very easily the details of elephant. When a circus company comes to your city and take elephants in procession, then the child if observe the procession, it will immediately recognize the elephant. This is the exact use of a model. In your classrooms your teacher will explain various aspects of the subject by drawing neat sketches on the black board. You will understand very easily and when you come across real world system, you can apply what all you learnt in your classroom. Hence through a model, we can explain the aspect of the subject / problem / system. The inequalities given in section 1.8.5 is a mathematical model, which explains clearly the manufacturing system, given in section 1.8.1. (***Here we can say a model is a relationship among specified variables and parameters of the system***).

Classification of Models

The models we use in operations research may broadly classified as:
(*i*) Mathematical and Descriptive models, and (*ii*) Static and Dynamic Models.

Mathematical and Descriptive Models

(*i*) Descriptive Model

A descriptive model explains or gives a description of the system giving various variables, constraints and objective of the system or problem. In article 1.8.1 gives the statement of the problem, which is exactly a descriptive model. The drawback of this model is as we go on reading and proceed; it is very difficult to remember about the variables and constraints, in case the problem or description of the system is lengthy one. It is practically impossible to keep on reading, as the manager has to decide the course of action to be taken timely.

Hence these models, though necessary to understand the system, have limited use as far as operations research is concerned.

(*ii*) Mathematical Model

In article, 1.8.2 we have identified the variables and constraints and objective in the problem statement and given them mathematical symbols x and y and a model is built in the form of an inequality of type. Objective function is also given. This is exactly a mathematical model, which explains the entire system in mathematical language, and enables the operations research person to proceed towards solution.

Types of Models

Models are also categorized depending on the *structure, purpose, nature of environment, behaviour, by method of solution and by use of digital computers.*

(*a*) Classification by Structure

(*i*) *Iconic Models:* These models are scaled version of the actual object. For example a toy of a car is an iconic model of a real car. In your laboratory, you might have seen Internal Combustion Engine models, Boiler models etc. All these are iconic models of actual engine and boiler etc. They explain all the features of the actual object. In fact a globe is an iconic model of the earth. These models may be of enlarged version or reduced version. Big objects are scaled down (reduced version) and small objects, when we want to show the features, it is scaled up to a bigger version. In fact it is a descriptive model giving the description of various aspects of real object. As far as operations research is concerned, is of less use. The advantages of these models: are It is easy to work with an iconic model in some cases, these are easy to construct and these are useful in describing static or dynamic phenomenon at some definite time. The limitations are, we cannot study the changes in the operation of the system. For some type of systems, the model building is very costly. It will be sometimes very difficult to carry out experimental analysis on these models.

(*ii*) *Analogue Model:* In this model one set of properties are used to represent another set of properties. Say for example, blue colour generally represents water. Whenever we want to show water source on a map it is represented by blue colour. Contour lines on the map is also analog model. Many a time we represent various aspects on graph by defferent colours or different lines all these are analog models. These are also not much used in operations research. The best examples are warehousing problems and layout problems.

(*iii*) *Symbolic Models or Mathematical Models*: In these models the variables of a problem is represented by mathematical symbols, letters etc. To show the relationships between variables and constraints we use mathematical symbols. Hence these are known as symbolic models or mathematical models. These models are used very much in operations research. Examples of such models are Resource allocation model, Newspaper boy problem, transportation model etc.

(*b*) Classification by utility

Depending on the use of the model or purpose of the model, the models are classified as

D*escriptive, Predictive and Prescriptive models.*

(*i*) *Descriptive model*: The descriptive model simply explains certain aspects of the problem or situation or a system so that the user can make use for his analysis. It will not give full details and clear picture of the problem for the sake of scientific analysis.

(*ii*) *Predictive model*: These models basing on the data collected, can predict the approximate results of the situation under question. For example, basing on your performance in the examination and the discussions you have with your friends after the examination and by verification of answers of numerical examples, you can predict your score or results. This is one type of predictive model.

(*iii*) *Prescriptive models*: We have seen that predictive models predict the approximate results.

But if the predictions of these models are successful, then it can be used conveniently to prescribe the courses of action to be taken. In such case we call it as Prescriptive model. Prescriptive models prescribe the courses of action to be taken by the manager to achieve the desired goal.

(*c*) Classification by nature of environment

Depending on the environment in which the problem exists and the decisions are made, and depending on the conditions of variables, the models may be categorized as *Deterministic models* and *Probabilistic models*.

(*i*) *Deterministic Models*: In this model the operations research analyst assumes complete certainty about the values of the variables and the available resources and expects that they do not change during the planning horizon. All these are deterministic models and do not contain the element of uncertainty or probability. The problems we see in Linear Programming, assumes certainty regarding the values of variables and constraints hence the Linear Programming model is a Deterministic model.

(*ii*) *Probabilistic or Stochastic Models*: In these models, the values of variables, the pay offs of a certain course of action cannot be predicted accurately because of element of probability. It takes into consideration element of risk into consideration. The degree of certainty varies from situation to situation. A good example of this is the sale of insurance policies by Life Insurance Companies to its customers. Here the failure of life is highly probabilistic in nature. The models in which the pattern of events has been compiled in the form of probability distributions are known as Probabilistic or Stochastic Models.

(*d*) Classification depending on the behaviour of the problem variables

Depending on the behaviour of the variables and constraints of the problem they may be classified as *Static Models* or *Dynamic models*.

(*i*) *Static Models*: These models assumes that no changes in the values of variables given in the problem for the given planning horizon due to any change in the environment or conditions of the system. All the values given are independent of the time. Mostly, in static models, one decision is desirable for the given planning period.

(*ii*) *Dynamic Models*: In these models the values of given variables goes on changing with time or change in environment or change in the conditions of the given system. Generally, the dynamic models then exist a series of interdependent decisions during the planning period.

(*e*) Classification depending on the method of getting the solution

We may use different methods for getting the solution for a given model. Depending on these methods, the models are classified as A*nalytical Models* and *Simulation Models*.

(*i*) *Analytical Models:* The given model will have a well-defined mathematical structure and can be solved by the application of mathematical techniques. We see in our discussion that the Resource allocation model, Transportation model, Assignment model, Sequencing model etc. have well defined mathematical structure and can be solved by different mathematical techniques. For example, Resource allocation model can be solved by Graphical method or by Simplex method depending on the number of variables involved in the problem. All models having mathematical structure and can be solved by mathematical methods are known as Analytical Models.

(*ii*) *Simulation Models:* The meaning of simulation is *imitation*. These models have mathematical structure but cannot be solved by using mathematical techniques. It needs certain experimental analysis. To study the behaviour of the system, we use random numbers. More complex systems can be studied by simulation. Studying the behaviour of laboratory model, we can evaluate the required values in the system. Only disadvantage of this method is that it does not have general solution method.

Some of the Points to be Remembered while Building a Model

* When we can solve the situation with a simple model, do not try to build a complicated model.
* Build a model that can be easily fit in the techniques available. Do not try to search for a technique, which suit your model.
* In order to avoid complications while solving the problem, the fabrication stage of modeling must be conducted rigorously.
* Before implementing the model, it should be validated / tested properly.
* Use the model for which it is deduced. Do not use the model for the purpose for which it is not meant.
* Without having a clear idea for which the model is built do not use it. It is better before using the model; you consult an operations research analyst and take his guidance.
* Models cannot replace decision makers. It can guide them but it cannot make decisions. Do not be under the impression, that a model solves every type of problem.
* The model should be as accurate as possible.
* A model should be as simple as possible.
* Benefits of model are always associated with the process by which it is developed.

Advantages of a Good Model

(*i*) A model provides logical and systematic approach to the problem.
(*ii*) It provides the analyst a base for understanding the problem and think of methods of solving. (*iii*) The model will avoid the duplication work in solving the problem.
(*iv*) Models fix the limitation and scope of an activity.

(*v*) Models help the analyst to find newer ways of solving the problem. (*vi*) Models saves resources like money, time etc.

(*vii*) Model helps analyst to make complexities of a real environment simple.

(*viii*) Risk of tampering the real object is reduced, when a model of the real system is subjected to experimental analysis.

(*ix*) Models provide distilled economic descriptions and explanations of the operation of the system they represent.

Limitations of a Model

(*i*) Models are constructed only to understand the problem and attempt to solve the problem;
they are not to be considered as real problem or system.

(*ii*) The validity of any model can be verified by conducting the experimental analysis and with relevant data characteristics.

Characteristics of a Good Model

(*i*) The number of parameters considered in a model should be less to understand the problem easily.

(*ii*) A good model should be flexible to accommodate any necessary information during the stages of building the model.

(*iii*) A model must take less time to construct.

(*iv*) A model may be accompanied by lower and upper bounds of parametric values.

Steps in Constructing a Model

(*i*) *Problem environment analysis and formulation*: One has to study the system in all aspects, if necessary make relevant assumptions, have the decision for which he is constructing the model in mind and formulate the model.

(*ii*) *Model construction and assumptions*: Identify the main variables and constraints and relate them logically to arrive at a model.

(*iii*) *Testing the model*: After the formulation, before using check the model for its validity.

. Methods of Solving Operations Research Problems

There are three methods of solving an operations research problem. They are:

(*i*) Analytical method, (*ii*) Iterative Method, (*iii*) The Monte-Carlo Technique.

(*i*) *Analytical Method*: When we use mathematical techniques such as differential calculus, probability theory etc. to find the solution of a given operations research model, the method of solving is known as analytical method and the solution is known as analytical solution. Examples are problems of inventory models. This method evaluates alternative policies efficiently.

(*ii*) *Iterative Method (Numerical Methods)*: This is trial and error method. When we have large number of variables, and we cannot use classical methods successfully, we use iterative process. First, we set a trial solution and then go on changing the solution under a given set of conditions, until no more modification is possible. The characteristics of this method is that the trial and error method used is laborious, tedious, time consuming and costly. The solution we get may not be accurate one and is approximate one. Many a time we find that after certain number of iterations, the solution cannot be improved and we have to accept it as the expected optimal solution.

(*iii*) *Monte-Carlo Method*: This method is based on random sampling of variable's values from a distribution of the variable. This uses sampling technique. A table of random numbers must be available to solve the problems. In fact it is a simulation process.

LINEAR PROGRAMMING:

Linear Programming Model

This model is used for resource allocation when the resources are limited and there are number of competing candidates for the use of resources. The model may be used to maximise the returns or minimise the costs. Consider the following two situations:

(*a*) A company which is manufacturing variety of products by using available resources, want to use resources optimally and manufacture different quantities of each type of product, which yield different returns, so as to maximise the returns.

(*b*) A company manufactures different types of alloys by purchasing the three basic materials and it want to maintain a definite percentage of basic materials in each alloy. The basic materials are to be purchased from the sellers and mix them to produce the desired alloy. This is to be done at minimum cost.

Both of them are resource allocation models, the case (*a*) is maximisation problem and the case (*b*) is minimisation problem.

(*c*) Number of factories are manufacturing the same commodities in different capacities and the commodity is sent to various markets for meeting the demands of the consumers, when the cost of transportation is known, the linear programming helps us to formulate a programme to distribute the commodity from factories to markets at minimum cost. The model used is transportation model.

(*d*) When a company has number of orders on its schedule, which are to be processed on same machines and the processing time, is known, then we have to allocate the jobs or orders to the machines, so as to complete all the jobs in minimum time. This we can solve by using Assignment model.

All the above-discussed models are Linear Programming Models. They can be solved by application of appropriate models, which are linear programming models.

Sequencing Model

When a manufacturing firm has some job orders, which can be processed on two or three machines and the processing times of each job on each machine is known, then the problem of processing in a sequence to minimise the cost or time is known as Sequencing model.

3. Waiting Line Model or Queuing Model

A model used for solving a problem where certain service facilities have to provide service to its customers, so as to avoid lengthy waiting line or queue, so that customers will get satisfaction from effective service and idle time of service facilities are minimised is waiting line model or queuing model.

4. Replacement Model

Any capital item, which is continuously used for providing service or for producing the product is subjected to wear and tear due to usage, and its efficiency goes on reducing. This reduction in efficiency can be predicted by the increasing number of breakdowns or reduced productivity. The worn out parts or components are to be replaced to bring the machine back to work. This action is known as maintenance. A time is reached when the maintenance cost becomes very high and the manager feels to replace the old machine by new one. This type of problems known as replacement problems and can be solved by replacement models.

5. Inventory Models

Any manufacturing firm has to maintain stock of materials for its use. This stock of materials, which are maintained in stores, is known as inventory. Inventory is one form of capital or money. The company has to maintain inventory at optimal cost. There are different types of inventory problems, depending the availability and demand pattern of the materials. These can be solved by the application of inventory models.

In fact depending on the number of variables, characteristics of variables, and the nature of constraints different models are available. These models, we study when we go chapter wise.

ORIGIN, CHARACTERISTICS/FEATURES OF OR,

MAIN PHASES OF OR-

A model, which is used for optimum allocation of scarce or limited resources to competing products or activities under such assumptions as certainty, linearity, fixed technology, and constant profit per unit, is *linear programming*.

Linear Programming is one of the most versatile, powerful and useful techniques for making managerial decisions. Linear programming technique may be used for solving broad range of problems arising in business, government, industry, hospitals, libraries, etc. Whenever we want to allocate the available limited resources for various competing activities for achieving our desired objective, the technique that helps us is **LINEAR PROGRAMMING**. As a decision making tool, it has demonstrated its value in various fields such as production, finance, marketing, research and development and personnel management. Determination of optimal product mix (a combination of products, which gives maximum profit), transportation schedules, Assignment problem and many more. In this chapter, let us discuss about various types of linear programming models.

PROPERTIES OF LINEAR PROGRAMMING MODEL

Any linear programming model (problem) must have the following properties:

(*a*) The relationship between variables and constraints must be linear. (*b*) The model must have an objective function.

(*c*) The model must have structural constraints.

(*d*) The model must have non-negativity constraint.

Let us consider a product mix problem and see the applicability of the above properties.

Example 2.1. A company manufactures two products *X* and *Y*, which require, the following resources. The resources are the capacities machine M_1, M_2, and M_3. The available capacities are 50,25,and 15 hours respectively in the planning period. Product *X* requires 1 hour of machine M_2 and 1 hour of machine M_3. Product *Y* requires 2 hours of machine M_1, 2 hours of machine M_2 and 1 hour of machine M_3. The profit contribution of products *X* and *Y* are Rs.5/- and Rs.4/- respectively.

The contents of the statement of the problem can be summarized as follows:

Machines	*Products*		*Availability in hours*
	X	*Y*	
M_1	0	2	50
M_2	1	2	25
M_3	1	1	15
Profit in Rs. Per unit	5	4	

In the above problem, Products *X* and *Y* are competing candidates or variables.

Machine capacities are available resources. Profit contribution of products *X* and *Y* are given. Now let us formulate the model.

Let the company manufactures *x* units of *X* and y units of *Y*. As the profit contributions of *X* and *Y* are Rs.5/- and Rs. 4/- respectively. The objective of the problem is to maximize the profit *Z*, hence objective function is:

Maximize $Z = 5x + 4y$ ⟶ OBJECTIVE FUNCTION.

This should be done so that the utilization of machine hours by products x and y should not exceed the available capacity. This can be shown as follows:

For Machine M_1 $0x + 2y$ 50
For Machine M_2 $1x + 2y$ 25 and
For machine M_3 $1x + 1y$ 15

⟶ **LINEAR STRUCTURAL CONSTRAINTS.**

But the company can stop production of *x* and *y* or can manufacture any amount of *x* and *y*. It cannot manufacture negative quantities of *x* and *y*. Hence we have write,

Both x and y are 0 . ⟶ NON -NEGATIVITY CONSTRAINT.

As the problem has got objective function, structural constraints, and non-negativity constraints and there exist a linear relationship between the variables and the constraints in the form of inequalities, the problem satisfies the properties of the Linear Programming Problem.

Basic Assumptions

The following are some important assumptions made in formulating a linear programming model:

1. It is assumed that the decision maker here is *completely certain* (*i.e.*, deterministic conditions) regarding all aspects of the situation, *i.e.*, availability of resources, profit contribution of the products, technology, courses of action and their consequences etc.
2. It is assumed that the relationship between variables in the problem and the resources available *i.e.*, constraints of the problem exhibits *linearity*. Here the term linearity implies proportionality and additivity. This assumption is very useful as it simplifies modeling of the problem.
3. We assume here *fixed technology*. Fixed technology refers to the fact that the production requirements are fixed during the planning period and will not change in the period.
4. It is assumed that the *profit contribution of a product remains constant*, irrespective of level of production and sales.

5. It is assumed that the decision variables are *continuous*. It means that the companies manufacture products in fractional units. For example, company manufacture 2.5 vehicles,

   3.2 barrels of oil etc. This is referred too as the assumption of *divisibility*.

6. It is assumed that *only one decision* is required for the planning period. This condition shows that the linear programming model is a static model, which implies that the linear programming problem is a *single stage decision problem*. (Note: Dynamic Programming problem is a multistage decision problem).

7. All variables are restricted to *nonnegative values* (*i.e*., their numerical value will be 0).

Terms Used in Linear Programming Problem

Linear programming is a method of obtaining an optimal solution or programme (say, product mix in a production problem), when we have limited resources and a good number of *competing candidates to consume* the limited resources in *certain proportion*. The term linear implies the condition of proportionality and additivity. The *programme* is referred as a course of action covering a specified period of time, say planning period. The manager has to find out the best course of action in the interest of the organization. This best course of action is termed as *optimal course of action or optimal solution* to the problem. A programme is optimal, when it *maximizes or minimizes* some measure or criterion of effectiveness, such as profit, sales or costs.

The term *programming* refers to a systematic procedure by which a particular program or plan of action is designed. Programming consists of a series of instructions and computational rules for solving a problem that can be worked out manually or can fed into the computer. In solving linear programming problem, we use a systematic method known as *simplex method* developed by American mathematician George B. Dantzig in the year 1947.

The candidates or activity here refers to number of products or any such items, which need the utilization of available resources in a certain required proportion. The available resources may be of any nature, such as money, area of land, machine hours, and man-hours or materials. But they are *limited in availability* and which are desired by the activities / products for consumption.

General Linear Programming Problem

A general mathematical way of representing a Linear Programming Problem (L.P.P.) is as given below:

$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ subjects to the conditions, OBJECTIVE FUNCTION

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1j}x_j + \dots + a_{1n}x_n$ (, =,) b_1

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots\dots + a_{2j}x_j + \dots\dots + a_{2n}x_n$ (, =,) b_2

Structural Constraints

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$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mj}x_j \dots + a_{mn}x_n$ (, =,) b_m

and all x_j are = 0 NON NEGETIVITY CONSTRINT. Where j = 1, 2, 3, … n

Where all c_j s, b_i s and a_{ij} s are constants and x_j s are decision variables.

To show the relationship between left hand side and right hand side the symbols , = , are used. Any one of the signs may appear in real problems. Generally sign is used for maximization problems and sign is used for minimization problems and in some problems, which are known as mixed problems we may have all the three signs. The word optimize in the above model indicates either maximise or minimize. The linear function, which is to be optimized, is the objective function. The inequality conditions shown are constraints of the problem. Finally all x_i s should be positive, hence the non-negativity function.

The steps for formulating the linear programming are:

1. ***Identify the unknown decision variables to be determined and assign symbols to them.***

2. ***Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.***

3. ***Identify the objective or aim and represent it also as a linear function of decision variables.***

Construct linear programming model for the following *problems*:

MAXIMIZATION MODELS

Example 2.2. A retail store stocks two types of shirts *A* and *B*. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type *A* and a maximum of 300 shirts of type *B*. The storage capacity, however, is limited to a maximum of

600 of both types combined. Type *A* shirt fetches a profit of Rs. 2/- per unit and type *B* a profit of Rs. 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem.

Solution: Here shirts *A* and *B* are problem variables. Let the store stock '*a*' units of *A* and '*b*' units of

B. As the profit contribution of *A* and *B* are Rs.2/- and Rs.5/- respectively, objective function is: Maximize $Z = 2a + 5b$ subjected to condition (s.t.)

Structural constraints are, stores can sell 400 units of shirt *A* and 300 units of shirt *B* and the storage capacity of both put together is 600 units. Hence the structural constraints are:

1*a* + 0*b* 400 and 0*a* + 1*b* 300 for sales capacity and 1*a* + 1*b* 600 for storage capacity. And non-negativity constraint is both *a* and *b* are 0. Hence the model is:

Maximize $Z = 2a + 5b$ s.t.

1*a* + 400
0*a* + 300
1*a* + 600
Both *a* and *b* are 0.

Problem 2.3. A ship has three cargo holds, forward, aft and center. The capacity limits are: Forward 2000 tons, 100,000 cubic meters

Center 3000 tons, 135,000 cubic meters

Aft 1500 tons, 30,000 cubic meters.

The following cargoes are offered, the ship owners may accept all or any part of each commodity:

Commodity	*Amount in tons.*	*Volume/ton in cubic*	*Profit per ton in Rs.*
A	6000	60	60
B	4000	50	80
C	2000	25	50

In order to preserve the trim of the ship the weight in each hold must be proportional to the capacity in tons. How should the cargo be distributed so as to maximize profit? Formulate this as linear programming problem.

Solution: Problem variables are commodities, *A*, *B*, and *C*. Let the shipping company ship '*a*' units of

A and '*b'* units of *B* and '*c'* units of *C*. Then Objective function is: Maximize $Z = 60a + 80b + 50c$ s.t.

Constraints are:

Weight constraint: $6000a + 4000b + 2000c$ 6,500 (= 2000+3000+1500)

The tonnage of commodity is 6000 and each ton occupies 60 cubic meters, hence there are 100 cubic meters capacity is available.

Similarly, availability of commodities *B* and *C*, which are having 80 cubic meter capacities each. Hence capacity inequality will be:

$100a + 80b + 80c$ 2,65,000 (= 100,000+135,000+30,000). Hence the l.p.p.

Model is: Maximise $Z = 60a+80b+50c$ s.t. $100a = 6000/60 = 100$

$6000a + 4000b + 2000c$ 6,500 $80b = 4000/50 = 80$

$100a+80b+80c$ 2,65,000 and $80c = 2000/25 = 80$ etc.

a,*b*,*c* all   0

MINIMIZATION MODELS

Problem 2.4. A patient consult a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins, vitamin *A* and vitamin *D*. Doctor advises him to consume vitamin *A* and *D* regularly for a period of time so that he can regain his health. Doctor prescribes tonic *X* and tonic *Y*, which are having vitamin *A*, and *D* in certain proportion. Also advises the patient to consume **at least** 40 units of vitamin *A* and 50 units of vitamin Daily. The cost of tonics *X* and *Y* and the proportion of vitamin *A* and *D* that present in *X* and *Y* are given in the table below. Formulate l.p.p. to minimize the cost of tonics.

Vitamins	*Tonic*		*Daily requirement in*
	X	*Y*	
A	2	4	40
D	3	2	50
Cost in Rs. per unit.	5	3	

Solution: Let patient purchase x units of X and y units of Y.

Objective function: Minimize $Z = 5x + 3y$

Inequality for vitamin A is $2x + 4y \quad 40$ (Here **at least** word indicates that the patient can consume more than 40 units but not less than 40 units of vitamin A daily).

Similarly the inequality for vitamin D is $3x + 2y \quad 50$.

For non–negativity constraint the patient cannot consume negative units. Hence both x and y

must be   0.

Now the l.p.p. model for the problem is:

Minimize $Z = 5x + 3y$ s.t.

$2x + 4y \quad 40$

$3x + 2y \quad 50$ and

Both x and y are   0.

Problem 2.5. A machine tool company conducts a job-training programme at a ratio of one for every ten trainees. The training programme lasts for one month. From past experience it has been found that out of 10 trainees hired, only seven complete the programme successfully. (The unsuccessful trainees are released). Trained machinists are also needed for machining. The company's requirement for the next three months is as follows:

January: 100 machinists, February: 150 machinists and March: 200 machinists.

In addition, the company requires 250 trained machinists by April. There are 130 trained machinists available at the beginning of the year. Pay roll cost per month is:

Each trainee Rs. 400/- per month.

Each trained machinist (machining or teaching): Rs. 700/- p.m. Each trained machinist who is idle: Rs.500/- p.m.

(Labour union forbids ousting trained machinists). Build a l.p.p. for produce the minimum cost hiring and training schedule and meet the company's requirement.

Solution: There are three options for trained machinists as per the data given. (*i*) A trained machinist can work on machine, (*ii*) he can teach or (*iii*) he can remain idle. It is given that the number of trained machinists available for machining is fixed. Hence the unknown decision variables are the number of machinists goes for teaching and those who remain idle for each month. Let,

'*a*' be the trained machinists teaching in the month of January.

'*b*' be the trained machinists idle in the month of January.

'*c'* be the trained machinists for teaching in the month of February.

'*d'* be the trained machinists remain idle in February.

'*e'* be the trained machinists for teaching in March.

'*f*' be the trained machinists remain idle in the month of March.

The constraints can be formulated by the rule that the number of machinists used for (machining + teaching + idle) = Number of trained machinists available at the beginning of the month.

For January $100 + 1a + 1b \quad 130$

For February, $150 + 1c + 1d = 130 + 7a$ (Here $7a$ indicates that the number of machinist trained is $10 \times a = 10a$. But only 7 of them are successfully completed the training *i.e.* $7a$).

For the month of March, $200 + 1e + 1f \quad 130 + 7a + 7c$

The requirement of trained machinists in the month of April is 250, the constraints for this will be

$130 + 7a + 7c + 7e \quad 250$ and the objective function is

Minimize $Z = 400\ (10a + 10c + 10e) + 700\ (1a + 1c + 1e) + 400\ (1b + 1d + 1f)$ and the non- negativity constraint is a, b, c, d, e, f all   0. The required model is:

Minimize $Z = 400\ (10a + 10c + 10e) + 700\ (1a + 1c + 1e) + 400\ (1b + 1d + 1f)$ s.t.

$100 + 1a + 1b \quad 130$

$150 + 1c + 1d \quad 130 + 7a$

$00 + 1e + 1f \quad 130 + 7a + 7c$

$130 + 7a + 7c + 7e \quad 250$ and

a, b, c, d, e, f all   0.

METHODS FOR THE SOLUTION OF A LINEAR PROGRAMMING PROBLEM

Linear Programming, is a method of solving the type of problem in which two or more **candidates** or **activities** are competing to utilize the available limited resources, with a view to **optimize** the **objective function** of the problem. The objective may be to maximize the **returns** or to minimize the **costs**. The various methods available to solve the problem are:

1. The Graphical Method when we have two decision variables in the problem. (To deal with more decision variables by graphical method will become complicated, because we have to deal with planes instead of straight lines. Hence in graphical method let us limit ourselves to two variable problems.
2. The Systematic Trial and Error method, where we go on giving various values to variables until we get optimal solution. This method takes too much of time and laborious, hence this method is not discussed here.
3. The Vector method. In this method each decision variable is considered as a vector and principles of vector algebra is used to get the optimal solution. This method is also time consuming, hence it is not discussed here.
4. The Simplex method. When the problem is having more than two decision variables, simplex method is the most powerful method to solve the problem. It has a systematic programme, which can be used to solve the problem.

One problem with two variables is solved by using both graphical and simplex method, so as to enable the reader to understand the relationship between the two.

2.5.1.Graphical Method

In graphical method, the inequalities (structural constraints) are considered to be equations. This is because; one cannot draw a graph for inequality. Only two variable problems are considered, because we can draw straight lines in two-dimensional plane (*X*- axis and *Y*-axis). More over as we have non- negativity constraint in the problem that is all the decision variables must have positive values always the solution to the problem lies in first quadrant of the graph. Some times the value of variables may fall in quadrants other than the first quadrant. In such cases, the line joining the values of the variables must be extended in to the first quadrant. The procedure of the method will be explained in detail while solving a numerical problem. The characteristics of Graphical method are:

(*i*) Generally the method is used to solve the problem, when it involves two decision variables. (*ii*) For three or more decision variables, the graph deals with planes and requires high imagination

to identify the solution area.

(*iii*) Always, the solution to the problem lies in first quadrant.

(*iv*) This method provides a basis for understanding the other methods of solution.

Problem 2.6. A company manufactures two products, *X* and *Y* by using three machines *A*, *B*, and *C*. Machine *A* has 4 hours of capacity available during the coming week. Similarly, the available capacity of machines *B* and *C* during the coming week is 24 hours and 35 hours respectively. One unit of product *X* requires one hour of Machine *A*, 3 hours of machine *B* and 10 hours of machine *C*. Similarly one unit of product *Y* requires 1 hour, 8 hour and 7 hours of machine *A*, *B* and *C* respectively. When one unit of *X* is sold in the market, it yields a profit of Rs. 5/- per product and that of *Y* is Rs. 7/- per unit. Solve the problem by using graphical method to find the optimal product mix.

Solution: The details given in the problem is given in the table below:

Machines	*Product*		*Available capacity in*
	X	*Y*	
A	1	1	4
B	3	8	24
C	10	7	35
Profit per unit in Rs.	5	7	

Let the company manufactures *x* units of *X* and y units of *Y*, and then the L.P. model is: Maximise $Z = 5x + 7y$ s.t.

$1x + 1y$ 4
$3x + 8y$ 24
$10x$ + $7y$ 35 and
Both *x* and *y* are   0.

As we cannot draw graph for inequalities, let us consider them as equations. Maximise $Z = 5x + 7y$ s.t.

$1x + 1y = 4$

$3x + 8y = 24$

$10x + 7y = 35$ and both *x* and *y* are   0

Problems to Solve

a. 1.An aviation fuel manufacturer sells two types of fuel A and B. Type A fuel is 25 % grade 1 gasoline, 25 % of grade 2 gasoline and 50 % of grade 3 gasoline. Type B fuel is 50 % of grade 2 gasoline and 50 % of grade 3 gasoline. Available for production are 500 liters per hour grade 1 and 200 liters per hour of grade 2 and grade 3 each. Costs are 60 paise per liter for grade 1, 120 paise for grade 2 and100 paise for grade 3. Type A can be sold at Rs. 7.50 per liter and B can be sold at Rs. 9.00 per liter. How much of each fuel should be made and sold to maximise the profit.

2. A company manufactures two products X_1 and X_2 on three machines A, B, and C. X_1 require
   a. 1 hour on machine A and 1 hour on machine B and yields a revenue of Rs.3/-. Product X_2
   b. requires 2 hours on machine A and 1 hour on machine B and 1 hour on machine C and yields
   c. revenue of Rs. 5/-. In the coming planning period the available time of three machines A, B, and C are 2000 hours, 1500 hours and 600 hours respectively. Find the optimal product mix.
3. A firm manufactures two types of products A and B and sells them at a profit of Rs.2 on type A and Rs.3 on type B. Each product is processed on two machines G and H. Type A requires one minute of processing time on G and 2 minutes on H, type B requires one minute on G and one minute on H. The machine G is available for not more than 6 hours and 40 minutes while machine H is available for 10 hours during one working day. Formulate the problem as a linear programming problem.

4. A furniture manufacturer makes two types of sofas — sofa of type A and sofa of type B. For simplicity, divide the production process into three distinct operations, say carpentry, finishing and upholstery. The amount of labor required for each operation Varies. Manufacture of a sofa of type A requires 6 hours of carpentry, 1 hour of finishing and 2 hours of upholstery. Manufacture of a sofa of type B requires 3 hours of carpentry, 1 hour of finishing and 6 hours of upholstery. Owing to limited availability of skilled labor as well as of tools and equipment, the factory has available each day 96 man hours of carpentry, 18 man hours for finishing and 72 man hours for upholstery. The profit per sofa of type A is Rs.80 and the profit per sofa of type B is Rs.70. How many sofas of type A and type B should be produced each day in order to maximize the profit? Formulate the problems as linear programming problem.

5. A furniture dealer deals in only two items, tables and chairs. He has Rs.5000/- to invest and a space to store at most 60 pieces. A table costs him Rs.250/- and a chair Rs.50/-. He can sell a table at a profit of Rs.50/- and a chair at a profit of Rs.15/-. Assuming, he can sell all the items that he buys, how should he invest his money in order that may maximize his profit?

MANAGERIAL APPLICATIONS,

LIMITATIONS OF OR. LINEAR PROGRAMMING:

FORMULATION OF LPP,

ASSUMPTIONS UNDERLYING LPP,

GRAPHICAL METHOD,

EXCEPTIONAL CASES- UNBOUNDED, INCONSISTENT SYSTEM OF CONSTRAINTS,

CONSISTENT AND NO SOLUTION, CONSTRAINTS ARE EQUATIONS.

UNIT II:. LPP - Simplex Method- Maximisation and Minimisation cases. Artificial variable technique - Big M method, Degeneracy. Dual - Formulation, Economic interpretation of dual - Sensitivity analysis.

LPP - SIMPLEX METHOD-

As discussed earlier, there are many methods to solve the Linear Programming Problem, such as Graphical Method, Trial and Error method, Vector method and Simplex Method. Though we use graphical method for solution when we have two problem variables, the other method can be used when there are more than two decision variables in the problem. Among all the methods, **SIMPLEX METHOD** is most powerful method. It deals with iterative process, which consists of first designing a **Basic Feasible Solution** or a **Programme** and proceed towards the **OPTIMAL SOLUTION** and testing each feasible solution for **Optimality** to know whether the solution on hand is optimal or not. If not an optimal solution, redesign the programme, and test for optimality until the test confirms **OPTIMALITY.** Hence we can say that the Simplex Method depends on two concepts known as **Feasibility** and **optimality**.

The simplex method is based on the property that the optimal solution to a linear programming problem, if it exists, can always be found in one of the basic feasible solution. The simplex method is quite simple and mechanical in nature. The iterative steps of the simplex method are repeated until a finite optimal solution, if exists, is found. If no optimal solution, the method indicates that no finite solution exists.

COMPARISION BETWEEN GRAPHICAL AND SIMPLEX METHODS

1. The graphical method is used when we have two decision variables in the problem. Whereas in Simplex method, the problem may have any number of decision variables.

2. In graphical method, the inequalities are assumed to be equations, so as to enable to draw straight lines. But in Simplex method, the inequalities are converted into equations by:

   (*i*) Adding a **SLACK VARIABLE** in maximisation problem and subtracting a **SURPLUS VARIABLE** in case of minimisation problem.

3. In graphical solution the **Isoprofit** line moves away from the origin to towards the far off point in maximisation problem and in minimisation problem, the **Isocost** line moves from far off distance towards origin to reach the nearest point to origin.

4. In graphical method, the areas outside the feasible area (area covered by all the lines of constraints in the problem) indicates idle capacity of resource where as in Simplex method, the presence of slack variable indicates the idle capacity of the resources.

5. In graphical solution, if the isoprofit line coincides with more than one point of the feasible polygon, then the problem has second alternate solution. In case of Simplex method the net- evaluation row has zero for non-basis variable the problem has alternate solution. (If two alternative optimum solutions can be obtained, the infinite number of optimum, solutions can be obtained).

However, as discussed in the forth coming discussion, the beauty of the simplex method lies in the fact that the relative exchange profitabilities of all the non -basis variables (vectors) can be determined simultaneously and easily; the replacement process is such that the new basis does not violate the feasibility of the solution.

Problems on Simplex Method

Simplex Method

1. A farmer has a 320 acre farm on which she plants two crops: corn and soybeans. For each acre of corn planted, her expenses are $50 and for each acre of soybeans planted, her expenses are $100. Each acre of corn requires 100 bushels of storage and yields a profit of $60; each acre of soybeans requires 40 bushels of storage and yields a profit of $90. If the total amount of storage space available is 19,200 bushels and the farmer has only $20,000 on hand, how many acres of each crop should she plant in order to maximize her profit? What will her profit be if she follows this strategy?

2. A retailer wishes to by a number of transistor radio sets of types A and B. Type A cost Rs.360/- each and type B cost Rs.240/- each. The retailer knows that he cannot sell more than 20 sets, so he does not want to buy more than 20 sets and he cannot afford to pay more than Rs.5760/-. His expectation is that he would get a profit of Rs.50/- for each set of type A and Rs.40/- for each set of type B. How many of each type should be purchased in order to make his total profit as large as Possible.

3. A soft drink company has two bottling plants, one located at P and the other at Q. Each plant produces three different soft drinks A, B, and C. The capacities of the two plants in number of bottles per day, are as follows:

	Plants	
Products	P	Q
A	3000	1000
B	1000	1000
C	2000	6000

A market survey indicates that during the month of May, there will be a demand for 24000 bottles of A, 16000 bottles of B and 48000 bottles of C. The operating cost per day of running
plants P and Q are respectively Rs.6000 and Rs.4000. How many days should the firm run each plant in the month of May so that the production cost is minimized.

ARTIFICIAL VARIABLE METHOD OR TWO PHASE METHOD

in linear programming problems sometimes we see that the constraints may have , or = signs. In such problems, basis matrix is not obtained as an identity matrix in the first simplex table; therefore, we introduce a new type of variable called, the artificial variable. These variables are fictitious and cannot have any physical meaning. The introduction of artificial variable is merely to get starting basic feasible solution, so that simplex procedure may be used as usual until the optimal solution is obtained. Artificial variable can be eliminated from the simplex table as and when they become zero *i.e*, non–basic. This process of eliminating artificial variable is performed in **PHASE I of the solution. PHASE II** is then used for getting optimal solution. Here the solution of the linear programming problem is completed in two phases, this method is known as **TWO PHASE SIMPLEX METHOD**. Hence, the two–phase method deals with removal of artificial variable in the fist phase and work for optimal solution in the second phase. If at the end of the first stage, there still remains artificial variable in the basic at a positive value, it means there is no feasible solution for the problem given. In that case, it is not necessary to work on phase II. If a feasible solution exists for the given problem, the value of objective function at the end of phase I will be zero and artificial variable will be non-basic. In phase II original objective coefficients are introduced in the final tableau of phase I and the objective function is optimized.

Problem 3.18: By using two phase method find whether the following problem has a feasible solution or not?

Maximize $Z = 4a + 5b$ s.t. Simplex version is: Max. $Z = 4a + 5b + 0S_1 + 0S_2 – MA$ s.t.

$2a + 4b \quad 8$ $\quad 2a + 4b + 1S_1 + 0S_2 + 0A = 8$

$1a + 3b \quad 9$ and both a and b are 0. $\quad 1a + 3b + 0S_1 – 1S_2 + 1A = 9$ and

a, b, S_1, S_2, A all are 0

DEGENERACY IN LINEAR PROGRAMMING PROBLEMS

The degeneracy in linear programming problems and the methods of solving degeneracy, if it exists, are discussed earlier in the chapter. To recollect the same a brief discussion is given below:

While improving the basic feasible solution to achieve optimal solution, we have to find the key column and key row. While doing so, we may come across two situations. One is **Tie** and the other is **Degeneracy**.

The tie occurs when two or more net evaluation row elements of variables are equal. In maximization problem, we select the highest positive element to indicate incoming variable and in minimization we select lowest element to indicate incoming variable (or highest numerical value with negative sign). When two or more net evaluation row elements are same, to break the tie, we select any one of them to indicate incoming variable and in the next iteration the problem of tie will be solved.

To select the out going variable, we have to select the lowest ratio or limiting ratio in the replacement ratio column. Here also, some times during the phases of solution, the ratios may be equal. This situation in linear programming problem is known as degeneracy. To solve degeneracy, the following methods are used:

1. Select any one row as you please. If you are lucky, you may get optimal solution, otherwise the problem cycles.

OR

2. Identify the rows, which are having same ratios. Say for example, S_1 and S_3 rows having equal ratio. In such case select the row, which contains the variable with smaller subscript. That is select row containing S_1 as the key row. Suppose the rows of variable x and z are having same ratio, then select the row-containing x as the key row.

3. (*a*) Divide the elements of unit matrix by corresponding elements of key column. Verify the ratios column-wise in unit matrix starting from left to right. Once the ratios are unequal, the degeneracy is solved. Select the minimum ratio and the row containing that element is the key row. (This should be done to the rows that are in tie).

(*b*) If the degeneracy is not solved by 3 (*a*), then divide the elements of the main matrix by the corresponding element in the key column, and verify the ratios. Once the ratios are unequal, select the lowest ratios. (This should be done only to rows that are in tie).

Problem 3.21: A company manufactures two product *A* and *B*. These are machined on machines *X* and *Y*. *A* takes one hour on machine *X* and one hour on Machine *Y*.

Similarly product *B* takes 4 hours on Machine *X* and 2 hours on Machine *Y*. Machine *X* and *Y* have 8 hours and 4 hours as idle capacity.

The planning manager wants to avail the idle time to manufacture *A* and *B*. The profit contribution of

A is Rs. 3/– per unit and that of *B* is Rs.9/– per unit. Find the optimal product mix.

Solution:

Maximize $Z = 3a + 9b$ s.t.

$1a + 4b \quad 8$

$1a + 2b \quad 4$ both *a* and *b* are $\quad 0$

Simplex format is:

Maximize $Z = 3a + 9b + 0S_1 + 0S_2$ s.t.

$1a + 4b + 1S_1 + 0S_2 = 8$

$1a + 2b + 0S_1 + 1S_2 = 4$ and

a, b, S_1, S_2 all $\quad 0$.

DUALITY IN LINEAR PROGRAMMING PROBLEMS

Most important finding in the development of Linear Programming Problems is the existence of **duality** in linear programming problems. Linear programming problems exist in pairs. That is in linear programming problem, every maximization problem is associated with a minimization problem. Conversely, associated with every minimization problem is a maximization problem. Once we have a problem with its objective function as maximization, we can write by using duality relationship of linear programming problems, its minimization version. The original linear programming problem is known as ***primal problem***, and the derived problem is known as dual problem.

The concept of the **dual problem** is important for several reasons. Most important are (*i*) the variables of dual problem can convey important information to managers in terms of formulating their future plans and (*ii*) in some cases the dual problem can be instrumental in arriving at the optimal solution to the original problem in many fewer iterations, which reduces the labour of computation.

Whenever, we solve the primal problem, may be maximization or minimization, we get the solution for the dual automatically. That is, the solution of the dual can be read from the final table of the primal and vice versa. Let us try to understand the concept of dual problem by means of an example. Let us consider the diet problem, which we have discussed while discussing the minimization case of the linear programming problem.

Example: The doctor advises a patient visited him that the patient is weak in his health due to shortage of two vitamins, *i.e.*, vitamin *X* and vitamin *Y*. He advises him to take at least 40 units of vitamin *X* and

50 units of Vitamin *Y* everyday. He also advises that these vitamins are available in two tonics *A* and *B*. Each unit of tonic *A* consists of 2 units of vitamin *X* and 3 units of vitamin *Y*. Each unit of tonic *B* consists of 4 units of vitamin *X* and 2 units of vitamin *Y*. Tonic *A* and *B* are available in the medical shop at a cost of Rs. 3 per unit of *A* and Rs. 2,50 per unit of *B*. The patient has to fulfill the need of vitamin by consuming *A* and *B* at a minimum cost.

The problem of **patient is the primal problem**. His problem is to minimize the cost. The tonics are available in the medical shop. The medical shop man wants to maximize the sales of vitamins A and B; hence he wants to maximize his returns by fixing the competitive prices to vitamins. The problem of medical shop person is the **dual problem. Note that the primal problem is minimization problem and the dual problem is the maximization problem.**

If we solve and get the solution of the primal problem, we can read the answer of dual problem from the primal solution.

Primal problem:

Minimize $Z = 3a + 2.5b$. s.t.

$2a + 4b \quad 40$

$3a + 2b \quad 50$

both a and b are 0.

Dual Problem:

Maximize $Z = 40x + 50y$ s.t.

$2x + 3y \quad 3$

$4x + 2y \quad 2.50$

both x and y are 0.

Solution to Primal: (Minimization problem *i.e.,* patient's problem)

Proble variabl e	Cos Rs.	C Requirement	3 a	2.50 b	0 p	0 q	M A 1	M A 2
b	2.5	5/2	0	1	–3/8	1/4	3/8	–1/4

a	3	15	1	0	1/4	–1/2	–1/4	1/2
		Net	0	0	3/16	7/8	M –	M – 7/8

Answer: *a* = 15 units, *b* = 2.5 units and total minimum cost is Rs. 51.25

Solution to Dual: (Maximization problem *i.e* medical shop man's problem)

Problem variable	*Profi Rs.*	*C Capacity units*	*40 x*	*50 y*	*0 S1*	*0 S*
y	50	7/8	0	1	1/2	–1/4
x	40	3/1	1	0	–1/4	3/8
		Net	0	0	–15	–

Answer: $x = 3/16$, $y = 7/8$, and maximum profit is Rs. 51.25

The patient has to minimize the cost by purchasing vitamin *X* and *Y* and the shopkeeper has to increase his returns by fixing competitive prices for vitamin *X* and *Y*. Minimum cost for patient is Rs. 51.25 and the maximum returns for the shopkeeper is Rs. 51.25. The competitive price for tonics is Rs.3 and Rs.2.50. Here we can understand the concept of **shadow price or economic worth of resources** clearly. If we multiply the original elements on the right hand side of the constraints with the net evaluation elements under slack or surplus variables we get the values equal to the minimum cost of minimization problem or maximum profit of the maximization problem. The concept of shadow price is similar to the economist's concept of the **wort**h of a marginal resource. In other words, we can see for a manufacturing unit it is **machine hour rate**. It is also known as **imputed value of the resources.** One cannot earn more than the economic worth of the resources he has on his hand. **The fact that the value of the objective function in the optimal program equals to the imputed value of the available resources has been called the FUNDAMENTAL THEOREM OF LINEAR PROGRAMMING.**

By changing the rows of the primal problem (dual problem) into columns we get the dual problem (primal problem) and vice versa.

To understand the rationale of dual problem and primal problem, let us consider another example.

Problem 3.32: A company manufactures two products *X* and *Y* on three machines Turning, Milling and finishing machines. Each unit of *X* takes, 10 hours of turning machine capacity, 5 hours of milling machine capacity and 1 hour of finishing machine capacity. One unit of *Y* takes 6 hours of turning machine capacity, 10 hours of milling machine capacity and 2 hours of finishing machine capacity. The company has 2500 hours of turning machine capacity, 2000 hours of milling machine capacity and 500 hours of finishing machine capacity in the coming planning period. The profit contribution of product *X* and *Y* are Rs. 23 per unit and Rs. 32 per unit respectively. Formulate the linear programming problems and write the dual.

Solution:

Department	Pro duct		Available
	X	*Y*	
Turning	10	6	250
Milling	5	10	200
Finishing	1	2	50
Profit per unit in	23	32	

Let us take the maximization problem stated to be primal problem. Associated with this maximization problem is the minimization problem that is the dual of the given primal problem. Let us try to formulate the dual by logical argument.

The primal problem is the **seller's maximization problem**, as the seller wants to maximize his profit. Now the technology, *i.e.*, the machinery required are with the seller and they are his available resources. Hence he has to prepare the plans to produce the products to derive certain profit and he wants to know what will be his profit he can get by using the available resources. Hence the buyer's problem is:

Maximize $23x + 32y$ s.t

$10x + 6y \quad 2500$

$5x + 10y \quad 2000$

$1x + 2y \quad 500$ and both x and $y \quad 0$ (This we shall consider as **primal** problem).

Associated with seller's maximization problem is a **buyer's minimization problem**. Let us assume seller will pretend to be the buyer. The rationale is the buyer, it is assumed, will consider the purchase of the resources in full knowledge of the technical specifications as given in the problem. If the buyer wishes to get an idea of his total outlay, he will have to determine how much must he pay to buy all the resources. Assume that he designates variables by *a*, *b*, *c*, to represent per unit price or value that he will assign to turning, milling and finishing capacities, respectively, while making his purchase plans. The total outlay, which the buyer wishes to minimize, will be determined by the function 2500*a* +

2000*b* + 500*c*, which will be the objective function of the buyer. The linear function of the objective function mentioned, must be minimized in view of the knowledge that the current technology yields a profit of Rs. 23 by spending 10 machine hours of turning department, 5 machine hours of milling department and 1 man-hour of finishing department. Similarly we can interpret other constraints also. Now the buyer's minimization problem will be:

Minimize. $Z = 2500a + 2000b + 500c$ s.t.

$10a + 5b + 1c \quad 23$

$6a + 10b + 2c \quad 32$ and all *a*, *b*, *c*, are 0 (This minimization version is the **dual of seller's primal problem given above**), where *a*, *b*, and *c* are dual variables and *x*, *y*, *z* are primal variables.

The values assigned to the dual variables in the optimal tableau of the dual problem, represent **artificial accounting prices, or implicit prices or shadow prices, or marginal worth or machine hour rate of various resources**. Because of this, we can read the values of dual variables from the net evaluation row of final tableau of primal problem. The values will be under slack variable column in net evaluation row.

The units of the constraint to which the dual variable corresponds determine the dimension of any dual variable.

In this problem the dimension of variable '*a*' as well as '*b*' is rupees per machine hour and that of variable '*c*' is rupees per man-hour.

Another important observation is by definition, the entire profit in the maximization must be traced to the given resources, the buyer's total outlay, at the equilibrium point, must equal to the total profit. That is, optimal of the objective function of the primal equals to the optimal value of the objective function of the dual. This observation will enable the problem solver to check whether his answer is correct or not. The total profit (cost) of maximization problem (minimization problem) must be equal to the shadow price (or economic worth) of resources.

The given **primal** problem will have **symmetrical dual**. The symmetrical dual means all given structural constraints are inequalities. All variables are restricted to nonnegative values.

Now let us write primal and dual side by side to have a clear idea about both.

Primal problem:

Maximize $Z = 23x + 32y$ s.t.

$10x + 6y \quad 2500$

$5x + 10y \quad 2000$

$1x + 2y \quad 500$

Both x and y are 0.

Dual Problem:

Minimize $Z = 2500a + 2000b + 5000c$ s.t.

$10a + 5b + 1c \quad 23$

$6a + 10b + 2c \quad 32$

All a, b, c, are 0.

Now let us discuss some of the important points that are to be remembered while dealing with primal and dual problem. Hence the characteristics are:

Note:

1. **If in the primal, the objective function is to be maximized, then in the dual it is to be minimized.**

   Conversely, if in the primal the objective function is to be minimized, then in the dual it is to be maximized.

2. **The objective function coefficients of the prima appear as right-hand side numbers in the dual and vice versa.**

3. **The right hand side elements of the primal appear as objective function coefficients in the dual and vice versa.**

4. **The input - output coefficient matrix of the dual is the transpose of the input – output coefficient matrix of the primal and vice versa.**

5. **If the inequalities in the primal are of the “less than or equal to” type then in the dual they are of the “greater than or equal to” type. Conversely, if the inequalities in the primal are of the “greater than or equal to” type; then in the dual they are of the “less than or equal to” type.**

6. **The necessary and sufficient condition for any linear programming problem and its dual to have optimum solution is that both have feasible solution. Moreover if one of them has a finite optimum solution, the other also has a finite optimum solution. The solution of the other (dual or primal) can be read from the net evaluation row (elements under slack/surplus variable column in net evaluation row). Then the values of dual variables are called *shadow prices*.**

7. If the primal (either) problem has an unbound solution, then the dual has no solution.

8. If the *i* th dual constraints are multiplied by –1, then *i* th primal variable computed from net evaluation row of the dual problem must be multiplied by –1.

9. If the dual has no feasible solution, then the primal also admits no feasible solution.

10. If *k* th constraint of the primal is equality, then the *k* th dual variable is unrestricted in sign.

11. If *p* th variable of the primal is unrestricted in sign, then the *p* th constraint of the dual is a strict equality.

Summary:

Primal	Dual
(*a*) Maximize.	Minimize
(*b*) Objective Function.	Right hand side.
(*c*) Right hand side.	Objective function.
(*d*) *i* th row of input-output	*i* th column of input output
(*e*) *j* th column of input-output	*j* the row of input-output
(*f*) *i* th relation of inequality ( ).	*i* th variable non-negative.
(*g*) *i* th relation is an equality (=).	*i* the variable is unrestricted in
(*h*) *j* th variable non-negative.	*j* relation an inequality ( ).
i th variable unrestricted in sign	*i* th relation an equality.

Note:

1. **Primal of a Prima is Primal**
2. **Dual of a Dual is Primal.**
3. **Primal of a Dual is Primal.**
4. **Dual of a Primal is Dual.**
5. **Dual of a Dual of a Dual is Primal.**

Procedure for converting a primal into a dual and vice versa

Case 1: When the given problem is maximization one:

The objective function of primal is of maximization type and the structural constraints are of type. Now if the basis variables are x, y and z, give different name for variables of dual. Let them be a, b, and c etc. Now write the structural constraints of dual reading column wise. The coefficients of variables in objective function will now become the left hand side constants of structural constraints. And the left hand side constants of primal will now become the coefficients of variables of objective function of dual. Consider the example given below:

Primal	**Dual**
Maximize: $Z = 2x + 3y$ s.t.	Minimize: $Z = 10a + 12b$ s.t.
$1x + 3y \quad 10$	$1a + 2b \quad 2$
$2x + 4y \quad 12$ and	$3a + 4b \quad 3$ and
Both x and y are 0	both a and b are 0

Case 2: When the problem is of Minimization

MAXIMISATION AND MINIMISATION CASES.

ARTIFICIAL VARIABLE TECHNIQUE - BIG M METHOD,

DEGENERACY. DUAL - FORMULATION,

ECONOMIC INTERPRETATION OF DUAL - SENSITIVITY ANALYSIS.

UNIT III: TRANSPORTATION & ASSIGNMENT PROBLEM: **Transportation Problem (TP) - IBFS using northwest corner rule, Row and Column Minimum methods, Matrix minimum method (LCM) and Vogel's approximation method, Unbalanced TP, Degeneracy, Optimality Test – MODI method and Managerial applications. Assignment Problem (AP): Unbalanced AP, Hungarian method. Travelling salesman problem, Managerial applications of AP and TSP.**

TRANSPORTATION & ASSIGNMENT PROBLEM:

TRANSPORTATION PROBLEM (TP) –

IBFS USING NORTHWEST CORNER RULE,

ROW AND COLUMN MINIMUM METHODS,

MATRIX MINIMUM METHOD (LCM) AND VOGEL'S APPROXIMATION METHOD,

UNBALANCED TP,

DEGENERACY,

OPTIMALITY TEST –

MODI METHOD AND MANAGERIAL

APPLICATIONS. ASSIGNMENT PROBLEM (AP):

UNBALANCED AP, HUNGARIAN METHOD.

TRAVELLING SALESMAN PROBLEM,

MANAGERIAL APPLICATIONS OF AP AND TSP.

UNIT IV: NETWORK PROBLEMS & SIMULATION : Network problems : Network fundamentals - Fulkerson's Rule – Uses & Applications of PERT/CPM Techniques- CPM - Crashing simple problems - PERT, Disadvantages of network techniques - Simulation – meaning, types, monte-carlo simulation process.(only theory)

INTRODUCTION

Programme Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are two techniques that are widely used in planning and scheduling the large projects. A project is a combination of various activities. For example, Construction of a house can be considered as a project. Similarly, conducting a public meeting may also be considered as a project. In the above examples, construction of a house includes various activities such as searching for a suitable site, arranging the finance, purchase of materials, digging the foundation, construction of superstructure etc. Conducting a meeting includes, printing of invitation cards, distribution of cards, arrangement of platform, chairs for audience etc. In planning and scheduling the activities of large sized projects, the two network techniques — PERT and CPM — are used conveniently to estimate and evaluate the project completion time and control the resources to see that the project is completed within the stipulated time and at minimum possible cost. Many managers, who use the PERT and CPM techniques, have claimed that these techniques drastically reduce the project completion time. But it is wrong to think that network analysis is a solution to all bad management problems. In the present chapter, let us discuss how PERT and CPM are used to schedule the projects. Initially, projects were represented by **milestone chart** and **bar chart**. But they had little use in controlling the project activities. **Bar chart** simply represents each activity by bars of length equal to the time taken on a common time scale as shown in figure 15. l. This chart does not show interrelationship between activities. It is very difficult to show the progress of work in these charts. An improvement in bar charts is **milestone chart**. In milestone chart, key events of activities are identified and each activity is connected to its preceding and succeeding activities to show the logical relationship between activities. Here each key event is represented by a node (a circle) and arrows instead of bars represent activities, as shown in figure 15.2. The extension of milestone chart is PERT and CPM network methods.

PERT AND CPM

In PERT and CPM the milestones are represented as ***events***. Event or node is either starting of an activity or ending of an activity. Activity is represented by means of an arrow, which is resource consuming. Activity consumes resources like time, money and materials. Event will not consume any resource, but it simply represents either starting or ending of an activity. Event can also be represented by rectangles or triangles. When all activities and events in a project are connected logically and sequentially, they form a ***network***, which is the basic document in network-based management. The basic steps for writing a network are:

(*a*) List out all the activities involved in a project. Say, for example, in building construction, the activities are:

(*i*) Site selection,

(*ii*) Arrangement of Finance, (*iii*) Preparation of building plan,

(*iv*) Approval of plan by municipal authorities, (*v*) Purchase of materials,

(*vi*) Digging of foundation, (*vii*) Filling up of foundation, (*viii*) Building superstructure,

(*ix*) Fixing up of doorframes and window frames, (*x*) Roofing,

(*xi*) Plastering,
(*xii*) Flooring,

(*xiii*) Electricity and water fittings, (*xiv*) Finishing.

(*b*) Once the activities are listed, they are arranged in sequential manner and in logical order. For example, foundation digging should come before foundation filling and so on.

There are three time estimates in PERT, they are:

(*a*) **OPTIMISTIC TIME:** Optimistic time is represented by t_O. Here the estimator thinks that everything goes on well and he will not come across any sort of uncertainties and estimates lowest time as far as possible. He is optimistic in his thinking.

(*b*) **PESSIMISTIC TIME:** This is represented by t_P. Here estimator thinks that everything goes wrong and expects all sorts of uncertainties and estimates highest possible time. He is pessimistic in his thinking.

(*c*) **LIKELY TIME:** This is represented by t_L. This time is in between optimistic and pessimistic times. Here the estimator expects he may come across some sort of uncertainties and many a time the things will go right.

So while estimating the time for a PERT activity, the estimator will give the three time estimates. When these three estimates are plotted on a graph, the probability distribution that we get is closely associated with **Beta Distribution curve**. For a Beta distribution curve as shown in figure 6.10, the characteristics are:

Standard deviation = $(t_P - t_O)/6$ = , $t_P - t_O$ is known as range.

Variance = $\{(t_P - t_O)/6\}^2$ = 2

Expected Time or Average Time = $t_E = (t_O + 4t_L + t_P) / 6$

These equations are very important in the calculation of PERT times. Hence the student has to remember these formulae.

Now let us see how to deal with the PERT problems.

(*g*) **Numbering of events:** Once the network is drawn the events are to be numbered. In PERT network, as the activities are given in terms of events, we may not experience difficulty. Best in case of CPM network, as the activities are specified by their name, is we have to number the events. For numbering of events, we use D.R. Fulkerson's rule. As per this rule:

An initial event is an event, which has only outgoing arrows from it and no arrow enters it. Number that event as 1.

Delete all arrows coming from event 1. This will create at least one more initial event. Number these initial events as 2, 3 etc.

Delete all the outgoing arrows from the numbered element and which will create some more initial events. Number these events as discussed above.

Continue this until you reach the last event, which has only incoming arrows and no outgoing arrows.

While numbering, one should not use negative numbers and the initial event should not be assigned ‘zero’. When the project is considerably large, at the time of execution of the project, the project manager may come to know that some of the activities have been forgotten and they are to be shown in the current network. In such cases, if we use **skip numbering**, it will be helpful. Skip numbering means, skipping of some numbers and these numbers may be made use to represent the events forgotten. We can skip off numbers like 5, 10, 15 etc. or 10, 20 and 30 or 2, 12, 22 etc. Another way of numbering the network is to start with 10 and the second event is 20 and so on. This is a better way of numbering the events.

CRITICAL PATH METHOD (CPM) FOR CALCULATING PROJECT COMPLETION TIME

In critical path method, the time duration of activity is deterministic in nature *i.e.* there will be a single time, rather than three time estimates as in PERT networks. The network is activity oriented. The three ways in which the CPM type of networks differ from PERT networks are

CP	*PER*
(*a*) Network is constructed on the basis of jobs or activities (activity oriented)	(*a*) Network is constructed basing on the events
(*b*) CPM does not take uncertainties involved in the estimation of times. The time required is deterministic and	(*b*) PERT network deals with uncertainties and hence three time estimations are considered
(*c*) CPM times are related to cost. That is can be by decreasing the activity duration direct costs increased (crashing of activity duration is	(*c*) As there is no certainty of time, activity duration cannot be reduced. Hence cost cannot be expressed correctly. We can say

Simulation

INTRODUCTION

Simulation is the most important technique used in analyzing a number of complex systems where the methods discussed in previous chapters are not adequate. There are many real world problems which cannot be represented by a mathematical model due to stochastic nature of the problem, the complexity in problem formulation and many values of the variables are not known in advance and there is no easy way to find these values.

Simulation has become an important tool for tackling the complicated problem of managerial decision-making. Simulation determines the effect of a number of alternate policies without disturbing the real system. Recent advances in simulation methodologies, technical development and software availability have made simulation as one of the most widely and popularly accepted tool in Operation Research. Simulation is a quantitative technique that utilizes a computerized mathematical model in order to represent actual decision-making under conditions of uncertainty for evaluating alternative courses of action based upon facts and assumptions.

John Von Newmann and Stainslaw Ulam made first important application of simulation for determining the complicated behaviour of neutrons in a nuclear shielding problem, being too complicated for mathematical analysis. After the remarkable success of this technique on neutron problem, it has become more popular and has many applications in business and industry. The development of digital computers has further increased the rapid progress in the simulation technique.

Designers and analysts have long used the techniques of simulation by physical sciences. Simulation is the representative model of real situation. Fore example, in a city, a children's park with various signals and crossing is a simulated model of city traffic. A planetarium is a simulated model of the solar system. In laboratories we perform a number of experiments on simulated model to predict the behaviours of the real system under true environment. For training a pilot, flight simulators are used. The simulator under the control of computers gives the same readings as the pilot gets in real flight. The trainee can intervene when there is signal, like engine failure etc. Simulation is the process of generating values using random number without really conducting experiment. Whenever the experiments are costly or infeasible or time-consuming simulation is used to generate the required data.

DEFINITION

1. Simulation is a representation of reality through the use of model or other device, which will react in the same manner as reality under a given set of conditions.

2. Simulation is the use of system model that has the designed characteristic of reality in order to produce the essence of actual operation.

3. According to Donald G. Malcolm, simulation model may be defined as one which depicts the working of a large scale system of men, machines, materials and information operating over a period of time in a simulated environment of the actual real world conditions.

4. According to Naylor, et al. simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over extended period of time.

There are two types of simulation, they are:

1. Analog Simulation: Simulating the reality in physical form (*e.g.*: Children's park, planetarium, etc.) is known as analog simulation.

2. Computer Simulation: For problems of complex managerial decision-making, the analogue simulation may not be applicable. In such situation, the complex system is formulated into a mathematical model for which a computer programme is developed. Using high-speed computers then solves the problem. Such type of simulation is known as computer simulation or system simulation.

CLASSIFICATION OF SIMULATION MODELS

Simulation models are classified as:

(*a*) **Simulation of Deterministic models:**

In the case of these models, the input and output variables are not permitted to be random variables and models are described by exact functional relationship.

(*b*) **Simulation of Probabilistic models:**

In such cases method of random sampling is used. The techniques used for solving these models are termed as Monte-Carlo technique.

(*c*) **Simulation of Static Models:**

These models do not take variable time into consideration. (*d*) **Simulation of Dynamic Models:**

These models deal with time varying interaction.

ADVANTAGES OF SIMULATION

Simulation is a widely accepted technique of operations research due to the following reasons:

* It is straightforward and flexible.
* It can be used to analyze large and complex real world situations that cannot be solved by conventional quantitative analysis models.
* It is the only method sometimes available.
* It studies the interactive effect of individual components or variables in order to determine which ones are important.
* Simulation model, once constructed, may be used over and again to analyze all kinds of different situations.
* It is the valuable and convenient method of breaking down a complicated system into subsystems and their study. Each of these subsystems works individually or jointly with others.

LIMITATIONS OF SIMULATION TECHNIQUE

* Since simulation model mostly deals with uncertainties, the results of simulation are only reliable approximations involving statistical errors, optimum results cannot be produced by simulation.
* In many situations, it is not possible to identify all the variables, which affect the behaviour of the system.
* In very large and complex problems, it is very difficult to make the computer program in view of the large number of variables and the involved inter–relationship among them.
* For problems requiring the use of computer, simulation may be comparatively costlier and time consuming in many cases.
* Each solution model is unique and its solutions and inferences are not usually transferable to other problems, which can be solved by other techniques.

MONTE-CARLO SIMULATION

The Monte-Carlo method is a simulation technique in which statistical distribution functions are created using a series of random numbers. Working on the digital computer for a few minutes we can create data for months or years. The method is generally used to solve problems which cannot be adequately represented by mathematical models or where solution of the model is not possible by analytical method. Monte-Carlo simulation yields a solution, which should be very close to the optimal, but not necessarily the exact solution. But this technique yields a solution, which converges to the optimal solution as the number of simulated trials tends to infinity. The Monte-Carlo simulation procedure can be summarized in the following steps:

Step 1: Clearly define the problem:

(*a*) Identify the objectives of the problem.

(*b*) Identify the main factors, which have the greatest effect on the objective of the problem.

Step 2: Construct an approximate model:

(*a*) Specify the variables and parameters of the mode.

(*b*) Formulate the appropriate decision rules, *i.e.* state the conditions under which the experiment is to be performed.

(*c*) Identify the type of distribution that will be used. Models use either theoretical distributions or empirical distributions to state the patterns of the occurrence associated with the variables.

(*d*) Specify the manner in which time will change.

NETWORK PROBLEMS & SIMULATION :

NETWORK PROBLEMS : NETWORK FUNDAMENTALS –

FULKERSON'S RULE –

USES & APPLICATIONS OF PERT/CPM TECHNIQUES-

CPM - CRASHING SIMPLE PROBLEMS –

PERT, DISADVANTAGES OF NETWORK TECHNIQUES - SIMULATION – MEANING, TYPES, MONTE-CARLO SIMULATION PROCESS.(ONLY THEORY)

UNIT V: QUEUING THEORY & GAME THEORY : Queuing Theory– Concepts of Queue/Waiting line – General Structure of a Queuing System – Kendall's notation for representing queuing models – Simple problems on Single Channel Queuing Model – Poisson arrival and exponential service times with infinite population. Game Theory: characteristics, Concepts, Zero-sum game, two, three and more persons games, saddle point, optimal strategies, Value of the game, Principle of Dominance, Analytical method of solving two person zero sum games. – graphical solutions for (m x 2) and (2 x n) games

QUEUING THEORY & GAME THEORY :
QUEUING THEORY– CONCEPTS OF QUEUE/WAITING LINE –

Before going to ***waiting line theory or queuing theory****,* one has to understand two things in clear. They are ***service and customer or element.*** Here customer or element represents a person or machine or any other thing, which is in need of some service from servicing point. Service represents any type of attention to the customer to satisfy his need. For example,

1. Person going to hospital to get medical advice from the doctor is an element or a customer,
2. A person going to railway station or a bus station to purchase a ticket for the journey is a customer or an element,
3. A person at ticket counter of a cinema hall is an element or a customer,
4. A person at a grocery shop to purchase consumables is an element or a customer,
5. A bank pass book tendered to a bank clerk for withdrawal of money is an element or a customer,
6. A machine break down and waiting for the attention of a maintenance crew is an element or a customer.
7. Vehicles waiting at traffic signal are elements or customers,
8. A train waiting at outer signal for green signal is an element or a customer

Like this we can give thousands of examples. In the above cases, the service means,

1. Doctor is a service facility and medical care is a service,

2. Ticket counter is a service facility and issue of ticket is service.
3. Ticket counter is a service facility and issue of ticket is service.
4. Shop owner is a service facility and issue of items is service.
5. Bank clerk is a service facility and passing the cheque is service.
6. Maintenance crew is service facility and repairing the machine is service.
7. Traffic signals are service facility and control of traffic is service.
8. Signal post is a service facility and green signaling is service.

Above we have seen elements or customer and service facility and service. We can see here that all the customer or elements (hereafter called as customer only) will arrive and waits to avail the service at service station. When the service station has no desired capacity to serve them all at a time the customer has to wait for his/its chance resulting the formulation of a waiting line of customers which is generally known as a queue. In general we can say that ***a flow of customers***

HISTORICAL DEVELOPMENT OF THE THEORY

During 1903 Mr. A.K. Erlang, a Swedish engineer has started theoretical analysis of waiting line problem in telephone calls. In 1927, Mr. Millins developed the theory further and then by Mr. Thornton D Fry. But Mr. D.G.Kendall has given a systematic and mathematical approach to waiting line problem in

1951. After 1951 significant work has been done in waiting line theory, so as to enable it to apply to varieties of problems come across in industries and society. One best example of this may be quoted as the control of waiting time of a customer in queue complex of Tirupathi Temple. The present system of tying a belt with time to the hands of a customer is the results of application of queuing theory. Another example is computerized reservation of rail journey.

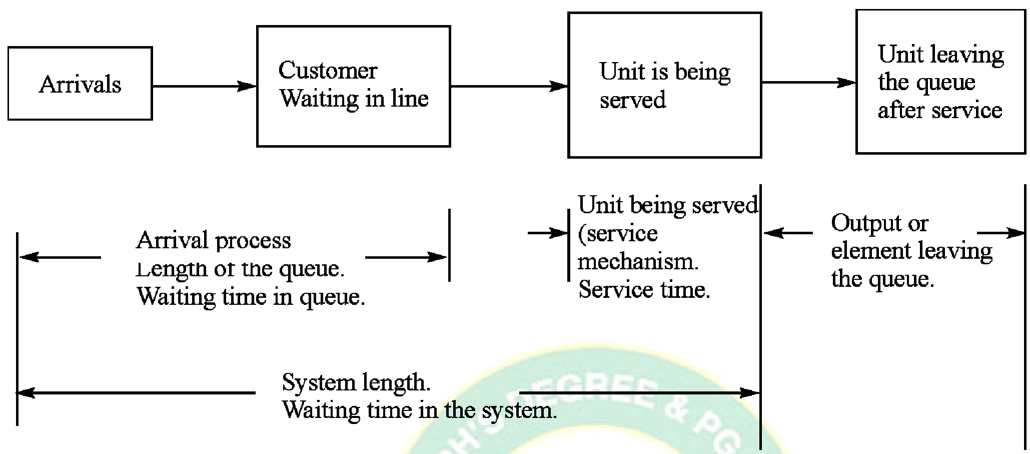
QUEUING SYSTEM OR PROCESS

One thing we have to remember is that when we speak of queue, we have to deal with two elements,

i.e. Arrivals and Service facility. Entire queuing system can be completely described by: (*a*) The input (Arrival pattern) (*b*) The service mechanism or service pattern, (*c*) The queue discipline and

(*d*) Customer behavior.

Components of the queuing system are arrivals, the element waiting in the queue, the unit being served, the service facility and the unit leaving the queue after service.



from infinite or finite population towards the service facility forms a queue or waiting line on account of lack of capability to serve them all at a time. The above discussion clarifies that the term customer we mean to the arriving unit that requires some service to be performed at the service station. **Queues** or **waiting lines** stands for a number of customers waiting to be serviced. Queue does not include the customer being serviced. The process or system that performs the services to the customer is termed as **service channel** or **service facility.**

Thus from the above we see that waiting lines or not only the lines formed by human beings but also the other things like railway coaches, vehicles, material etc.

A.K.Erlang, a Danish telephone engineer, did original work on queuing theory. Erlang started his work in 1905 in an attempt to determine the effects of fluctuating service demand (arrivals) on the utilization of automatic dialing equipment. It has been only since the end of World War II that work on waiting line models has been extended to other kinds of problems. In today's scenario a wide variety of seemingly diverse problems situations are recognized as being described by the general waiting line model. In any queuing system, we have an input that arrives at some facility for service or processing and the time between the arrivals of individual inputs at the service facility is commonly random in nature. Similarly, the time for service or processing is commonly a random variable.

QUEUING PROBLEMS

The most important information required to solve a waiting line problem is the nature and probability distribution of arrivals and service pattern. The answer to any waiting line problem depending on finding:

(*a*) *Queue length*: The probability distribution of queue length or the number of persons in the system at any point of time. Further we can estimate the probability that there is no queue.

(*b*) *Waiting time*: This is probability distribution of waiting time of customers in the queue. That is we have to find the time spent by a customer in the queue before the commencement of his service, which is called *his waiting time in the queue*. The total time spent in the system is the waiting time in the queue plus the service time. The waiting time depends on various factors, such as:

(*i*) The number of units already waiting in the system, (*ii*) The number of service stations in the system, (*iii*) The schedule in which units are selected for service,

(*iv*) The nature and magnitude of service being given to the element being served. (*c*) *Service time:* It is the time taken for serving a particular arrival.

(*d*) *Average idle time or Busy time distribution:* The average time for which the system remains idle. We can estimate the probability distribution of busy periods. If we suppose that the server is idle initially and the customer arrives, he will be provided service immediately. During his service time some more customers will arrive and will be served in their turn according to the system discipline. This process will continue in this way until no customer is left unserved and the server becomes free again after serving all the customers. At this stage we can conclude, that the busy period is over. On the other hand, during the idle periods no customer is present in the system. A busy period and the idle period following it together constitute a *busy cycle*. The study of busy period is of great interest in cases where technical features of the server and its capacity for continuous operation must be taken into account.

QUEUE MODELS

Most elementary queuing models assume that the *inputs / arrivals* and *outputs / departures* follow a **birth** and **death** process. Any queuing model is characterized by situations where both arrivals and departures take place simultaneously. Depending upon the nature of inputs and service faculties, there can be a number of queuing models as shown below:

(*i*) **Probabilistic queuing model:** Both arrival and service rates are some unknown random variables.

(*ii*) **Deterministic queuing model:** Both arrival and service rates are known and fixed.

(*iii*) **Mixed queuing model:** Either of the arrival and service rates is unknown random variable and other known and fixed.

Game Theory

INTRODUCTION

In previous chapters like Linear Programming, Waiting line model, Sequencing problem and Replacement model etc., we have seen the problems related to individual industrial concern and problems are solved to find out the decision variables which satisfy the objective of the industrial unit. But there are certain problems where two or more industrial units are involved in decision making under *conflict situation*. This means that decision-making is done to maximize the benefits and minimize the losses. The decision-making much depends on the decision made or decision variables chosen by the opponent business organization. Such situations are known as **competitive strategies**. Competitive strategies are a type of **business games**. When we here the word game, we get to our mind like pleasure giving games like Foot ball, Badminton, Chess, etc., In these games we have two parties or groups playing the game with definite well defined rules and regulations. The out come of the game as decided decides winning of a group earlier. In our discussion in Theory of Games, we are not concerned with pleasure giving games but we are concerned with **business games**. What is a business game?

Every business manager is interested in capturing the larger share in the market. To do this they have to use different strategies (course of action) to motivate the consumers to prefer their product. For example you might have seen in newspapers certain company is advertising for its product by giving a number of (say 10) eyes and names of 10 cine stars and identify the eyes of the stars and match the name with the eyes. After doing this the reader has to write why he likes the product of the company. For right entry they get a prize. This way they motivate the readers to prefer the product of the company. When the opponent company sees this, they also use similar strategy to motivate the potential market to prefer the product of their company. Like this the companies advertise in series and measure the growth in their market share. This type of game is known as **business game**. Managers competing for share of the market, army chief planning or execution of war, union leaders and management involved in collective bargaining uses different strategies to fulfill their objective or to win over the opponent. All these are known as business games or **competitive situation**. In business, competitive situations arise in advertising and marketing campaigns by competing business firms.

Hence, **Game theory is a body of knowledge that deals with making decisions when two or more intelligent and rational opponents are involved under conditions of conflict or competition. The competitors in the game are called *players.***

The beginning of **theory of games** goes back to 20 th century. But **John Von Neumann and Morgenstern** have mathematically dealt the theory and published a well-known paper **"theory of Games and Economic Behavior"** in 1944. The mathematical approach of Von Neumann utilizes the

Minimax principle, which involves the fundamental idea of **minimization of the maximum losses**. Many of the competitive problems can be handled by the game theory but not all the competitive problems can be analyzed with the game theory. Before we go to game theory, it is better for us to discuss briefly about decision-making.

DECISION MAKING

Making decision is an integral and continuous aspect of human life. For child or adult, man or woman, government official or business executive, worker or supervisor, participation in the process of decision- making is a common feature of everyday life. What does this process of decision making involve? What is a decision? How can we analyze and systematize the solving of certain types of decision problems? Answers of all such question are the subject matter of **decision theory**. Decision-making involves listing the various alternatives and evaluating them economically and select best among them. Two important stages in decision-making is: (*i*) making the decision and (*ii*) Implementation of the decision.

Analytical approach to decision making classifies decisions according to the amount and nature of the available information, which is to be fed as input data for a particular decision problems. Since future implementations are integral part of decision-making, available information is classified according to the degree of certainty or uncertainty expected in a particular future situation. With this criterion in mind, three types of decisions can be identified. First one is that these decisions are made when **future can be predicted with certainty.** In this case the decision maker assumes that there is only one possible future in conjunction with a particular course of action. The second one is that decision making under **conditions of risk**. In this case, the future can bring more than one state of affairs in conjunction with a specific course of action. The third one is decision making under **uncertainty**. In this case a particular course of action may face different possible futures, but the probability of such occurrence cannot be estimated objectively.

The Game theory models differ from **decision-making under certainty (DMUC)** and **decision- making under risk (DMUR)** models in two respects. First the opponent the decision maker in a game theory model is an active and rational opponent in DMUC and DMUR models the opponent is the passive state of nature. Second point of importance is decision criterion in game model is the **maximin** or the **minimax** criterion. In DMUC and DMUR models the criterion is the maximization or minimization of some measure of effectiveness such as profit or cost.

DESCRIPTION OF A GAME

In our day-to-day life we see many games like Chess, Poker, Football, Baseball etc. All these games are pleasure-giving games, which have the character of a competition and are played according to well- structured rules and regulations and end in a **victory** of one or the other team or group or a player. But we refer to the word **game** in this chapter the competition between two business organizations, which has more earning competitive situations. In this chapter game is described as:

A competitive situation is called a game if it has the following characteristics (Assumption made to define a game):

1. There is finite number of competitors called **Players**. This is to say that the game is played by two or more number of business houses. The game may be for creating new market, or to increase the market share or to increase the competitiveness of the product.

2. A list of finite or infinite number of possible **courses of action is available** to each player.

The list need not be the same for each player. Such a game is said to be in **normal form**. To explain this we can consider two business houses A and B. Suppose the player A has three strategies, as strategy I is to offer a car for the customer who is selected through advertising campaign. Strategy II may be a house at Ooty for the winning customer, and strategy III may a cash prize of Rs. 10,00,000 for the winning customer. This means to say that the competitor A has three strategies or courses of action. Similarly, the player B may have two strategies, for example strategy I is A pleasure trip to America for 10 days and strategy II may be offer to spend with a cricket star for two days. In this game A has three courses of action and B has two courses of actions. The game can be represented by mans of a matrix as shown below:

B

I
II

I
A
II

III

3. A play is played when each player chooses one of his courses o action. The choices are made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action. But in real world, a player makes the choices after the opponent has announced his course of action.

Every play *i.e.* combination of courses of action is associated with an out come, known as ***pay off*** - (generally money or some other quantitative measure for the satisfaction) which determines a set of gains, one to each player. Here **a loss is considered to be negative gain**. Thus after each playoff the game, one player pays to other an amount determined by the courses of action chosen. For example consider the following matrix:

		B		
		I	II	III
A	I	2	4	–3
	II	–1	2	2

In the given matrix, we have two players. Among these the player who is named on the left side matrix is known as winner, *i.e.* here A is the winner and the matrix given is the matrix of the winner. The player named above is known as the loser. The loser's matrix is the negative version of the given matrix. In the above matrix, which is the matrix of A, a winner, we can describe as follows. If A selects first strategy, and *B* selects the second strategy, the out come is +4 *i.e. A* will get 4 units of money and *B* loses 4 units of money. *i.e. B* has to give 4 units of money to *A*. Suppose *A* selects second strategy and *B* selects first strategy A's out come is –1, *i.e. A* loses one unit of money and he has to give that to *B*, it means *B* wins one unit of money.

4. All players act rationally and intelligently.

5. Each player is interested in **maximizing his gains or minimizing his losses**. The winner, *i.e.* the player on the left side of the matrix always tries to maximize his gains and is known as **Maximin player**. He is interested in maximizing his minimum gains. Similarly, the player B, who is at the top of the matrix, a loser always tries to minimize his losses and is known as **Minimax player** - *i.e.* who tries to minimize his maximum losses.

6. Each player makes individual decisions without direct communication between the players.

By principle we assume that the player play a strategy individually, without knowing opponent's strategy. But in real world situations, the player play strategy after knowing the opponent's choice to maximin or minimax his returns.

7. It is assumed that each player knows complete relevant information.

Game theory models can be classified in a number of ways, depending on such factors as the: (*i*) Number of players,

(*ii*) Algebraic sum of gains and losses

(*iii*) Number of strategies of each player, which decides the size of matrix.

Number of players: If number of players is two it is known as **Two-person game**. If the number of players is is '*n*' (where *n* 3) it is known as ***n*- person game**. In real world two person games are more popular. If the number of players is '*n*', it has to be reduced to two person game by two constant collations, and then we have to solve the game, this is because, the method of solving n- person games are not yet fully developed.

Algebraic sum of gains and losses: A game in which the gains of one player are the losses of other player or the algebraic sum of gains of both players is equal to zero, the game is known as **Zero sum game (ZSG)**. In a zero sum game the algebraic sum of the gains of all players after play is bound to be zero. i.e. If g_i as the pay of to a player in a n-person game, then the game will be a zero sum game if sum of all g_i is equal to zero.

In game theory, the resulting gains can easily be represented in the form of a matrix called **pay - off matrix** or **gain** matrix as discussed in S.No 3 above. A pay - off matrix is a table, which shows how payments should be made at end of a play or the game. Zero sum game is also known as **constant sum game**. Conversely, if the sum of gains and losses does not equal to zero, the game is a **nonzero**

-sum game. A game where two persons are playing the game and the sum of gains and losses is equal to zero, the game is known as **Two-Person Zero-Sum Game (TPZSG).** A good example of two- person game is the game of chess. A good example of n- person game is the situation when several companies are engaged in an intensive advertising campaign to capture a larger share of the market.

BASIC ELEMENTS OF GAME THEROY

Let us consider a game by name **Two-finger morra,** where two players (persons) namely *A* and *B* play the game. *A* is the winner and *B* is the loser. The matrix shown below is the matrix of *A*, the winner. The elements of the matrix show the gains of *A*. Any positive element in the matrix shows the gain of *A* and the negative element in the matrix show the loss (negative gain) of *A*.

The game is as follows: Both the players *A* and *B* sit at a table and simultaneously raise their hand with **one** or **two** fingers open. In case the fingers shown by both the players is same, then *A* will gain Rs.2/-. In case the number of fingers shown is different (*i.e.* A shows one finger and *B* shows two fingers or vice versa) then *A* has to give *B* Rs. 2/- *i.e. A* is losing Rs.2/-. In the above matrix, strategy I refer to finger one and strategy II refers to two fingers. The above given matrix is the pay of matrix of A. **The negative entries in the matrix denote the payments from *A* to *B*.** The pay of matrix of *B* is the negative version of *A*'s pay of matrix; because in two person zeros sum game the gains of one player are the losses of the other player. Always we have to write the matrix of the winner, who is represented on the left side of the matrix. The winner is **the maximizing player, who wants to maximize his minimum gains. The loser is the minimizing player, who wants to minimize his maximum losses.**

Note the following and remember

1. **The numbers within the payoff matrix represent the *outcome* or the *payoffs* of the different *plays* or *strategies* of the game. The payoffs are stated in terms of a measure of effectiveness such as money, percent of market share or utility.**

   By convention, in a 2-person, zero-sum game, the positive numbers denote a gain to the row or maximizing player or winner, and loss to the *column* or *minimizing player or loser*. It is assumed that both players know the payoff matrix.

2. A strategy is a course of action or a complete plan. It is assumed that a strategy cannot be upset by competitors or nature (chance). Each player may have any number of strategies. There is no pressure that both players must have same number of strategies.

3. *Rules of game* describe the framework within which player choose their strategies.

An assumption made here that *player must choose their strategies simultaneously and that the game is repetitive.*

4. A strategy is said to be *dominant* if each payoff in the strategy is *superior* to each *corresponding* pay off of alternative strategy. For example, let us consider *A* (winner) has three strategies. The payoffs of first strategy are 2, 1, 6 and that of second strategy are -1, -2 and 3. The second strategy's outcomes are inferior to that of first strategy. Hence first strategy dominates or superior to that of second strategy. Similarly let us assume B (loser) has two strategies. The outcomes of first strategy 2,

-1 and that of second strategy is 1 and -2. The payoffs of second strategy is better than that of first strategy, hence second strategy is superior and dominates the first strategy. The rule of dominance is used to reduce the size of the given matrix.

5. The *rule of game* refers to the expected outcome per play when both players follow their best or optimal strategies. A game is known as fair game if its value is zero, and unfair if its value is nonzero.

6. An *Optimal strategy* refers to the course of action, or complete plan, that leaves a player in the most preferred position regardless of the actions of his competitors. The meaning of the *most preferred position* is that any deviation from the optimal strategy, or plan, would result in decreased payoff.

7. The purpose of the game model is to identify the optimal strategy for each player.

The conditions said in serial number 1 to 3 above, the practical value of game theory is rather limited. However the idea of decision-making under conditions of conflict (or cooperation) is at the core of managerial decision. Hence the concepts involved in game theory are very important for the following reasons.

* It develops a framework for analyzing decision making in competitive (and sometimes in cooperative) situations. Such a framework is not available through any other analytical technique.
* It describes a systematic quantitative method (in two-person zero-sum games) that enables the competitors to select rational strategies for the attainment of their goals.

- **It describes and explains various phenomena in conflicting situations, such as bargaining and the formation of coalitions.**

THE TWO-PERSON, ZERO-SUM GAME: (Pure Strategy and Mixed Strategy games)

In our discussion, we discuss two types of Two-person, Zero-sum games. In one of the most preferred position for each player is achieved by adopting a **single strategy**. Hence this game is known as **pure- strategy game.** The second type requires the adoption by both players of **a mixture or a combination of** different strategies as opposed to a single strategy. Therefore this is termed as **mixed strategy game.**

In pure strategy game one knows, in advance of all plays that he will always choose only one particular course of action. **Thus pure strategy is a decision rule always to select the same course of action.** Every course of action is pure strategy.

A mixed strategy is that in which a player decides, in advance to choose on of his course of action in accordance with some fixed probability distribution. This in case of mixed strategy we associate probability to each course of action (each pure strategy). The pure strategies, which are used in mixed strategy game with non-zero probabilities, are termed as **supporting strategies**. Mathematically, a mixed strategy to any player is an ordered set of 'm' non-negative real numbers, which add to a sum unity (m is the number of pure strategies available to a player).

It is said above that in pure strategy game a player selects same strategy always, hence the opponent will know in advance the choice. But the superiority of mixed strategy game over pure strategy games is that the player is always kept guessing about the opponent's choice as innumerable combination of pure strategies one can adopt.

The purpose of the game theory is to determine the **best strategies** for each player on the basis of **maximin and minimax criterion of optimality. In this criterion a player lists his worst possible outcomes and then he chooses that strategy which corresponds to the best of those worst outcomes.** The **value of the game** is the maxim guaranteed gain to player. The value is denoted by **'v'**. The game whose value $v = 0$ is known as zero sum **game** or **fair game**. Solving the game mean to find the best strategies for both the players and find the value of the game.

The game theory does not insist on how a game should he played, but only tells the procedure and principles by which the action should be selected. Hence, **the game theory is a decision theory useful in competitive situations.** *The fundamental theorem assures that there exists a solution and the value of a rectangular game in terms of mixed strategies*.

CHARACTERISTICS OR PROPERTIES OF A GAME

To classify the games, we must know the properties of the game. They are: Number of persons or groups who are involved in playing the game

Number of strategies or courses of action each player or group have (they may be finite or infinite).

Type of course of action or strategy.

How much information about the past activities of other player is available to the players. It may be complete or partly or may be no information available.

The pay off may be such that the gains of some players may or may not be the direct losses of other players.

The players are independent in decision-making and they make the decision rationally.

The general rules of dominance can be formulated as below

1. **If all the elements of a column (say ith column) are greater than or equal to the corresponding elements of any other column (say *j*th column), then *i*th column is dominated by *j*th column.**
2. **If all the elements of *r*th row are less than or equal to the corresponding elements of any other row, say *s*th row, then *r*th row is dominated by *s*th row.**
3. **A pure strategy of a player may also be dominated if it is inferior to some convex combinations of two or more pure strategies, as a particular case, inferior to the averages of two or more pure strategies.**

Solutions to 2 x 2 games without saddle point: (Mixed strategies)

In rectangular games, when we have saddle point, the best strategies were the pure strategies. Now let us consider the games, which do not have saddle points. In such cases, the best strategies are the **mixed strategies**. While dealing with mixed strategies, we have to determine the probabilities with which each action should be selected. Let us consider a 2 × 2 game and get the formulae for finding the probabilities with which each strategy to be selected and the value of the game.

Points to be remembered in mixed strategy games are

(*a*) If one of the players adheres to his optimal mixed strategy and the other player deviates from his optimal strategy, then the deviating player can only decrease his yield and cannot increase in any case (at most may be equal).

(*b*) If one of the players adheres to is optimal strategy, then the value of the game does not alter if the opponent uses his supporting strategies only either singly or in any combination.

(*c*) If we add (or subtract) a fixed number say 1,to (from) each elements of the payoff matrix, then the optimal strategies remain unchanged while the value of the game increases (or decreases) by 1.

GENERAL STRUCTURE OF A QUEUING SYSTEM –

KENDALL'S NOTATION FOR REPRESENTING QUEUING MODELS –

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GAME THEORY: CHARACTERISTICS, CONCEPTS,

ZERO-SUM GAME, TWO, THREE AND MORE PERSONS GAMES,

SADDLE POINT,

OPTIMAL STRATEGIES,

VALUE OF THE GAME,

PRINCIPLE OF DOMINANCE,

ANALYTICAL METHOD OF SOLVING TWO PERSON ZERO SUM GAMES. –

GRAPHICAL SOLUTIONS FOR (M X 2) AND (2 X N) GAMES