

must be atleast in the order of

$$p = \Delta(Px) = 0.527 \times 10^{-19}$$

\therefore mass of electron (m_0) = 9.11×10^{-31} &
momentum (P) = 0.527×10^{-19} is

relativistic.

Therefore, Using Einstein relativistic formula for the electron.

$$E^2 = p^2 c^2 + m_0^2 c^4$$

Here, $m_0 c^2$ of the electron is about the order of 0.511 mv. which means that the 2nd term is very ^{much} smaller than the first term in relativistic energy. Hence it is neglected.

$$\therefore E^2 = p^2 c^2$$

$$E = pc$$

$$E = 0.527 \times 10^{-19} \times 3 \times 10^8 \text{ J}$$

$$E = \frac{0.527 \times 3 \times 10^{-11}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\left(\because 1 \text{ J} = \frac{1 \text{ eV}}{1.6 \times 10^{-19}} \right)$$

which means that if the electron exist inside the nucleus energy must be in the order of 10 mev. but several other theories, radio active β decay emission of electron gives max. energy

of 3-4 meV.

∴ Thus electron cannot exist inside the nucleus.

For a case of proton or neutron

$$m_0 = 1.67 \times 10^{-27} \text{ kg}; \text{ momentum } p = mc$$

$$c = p/m.$$

$$c = 3 \times 10^8 \text{ m/sec.}$$

In this case kinetic energy $E = \frac{p^2}{2m}$.

$$= \frac{(0.527 \times 10^{-19})^2}{2 \times 1.67 \times 10^{-27}}$$

$$= \frac{(0.527 \times 10^{-19})}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{19}}$$

$$= 52 \text{ KeV}$$

This energy is smaller than the energy carried by the protons & neutrons.

Thus, both particles exist inside the nucleus.

∴ It is impossible to stay electron inside the nucleus and proton & neutron does not stay outside of the nucleus.

* Nuclear Models :-

The precise nature of forces acting in the nucleus is unknown. Hence, nuclear models are resorted for investigation and theoretical prediction of its properties. Such models may be based on :-

- (1) The extrinsic analysis the properties of atomic nuclei.

- (2) The electron shell at an atom these model called as liquid drop model (or) shell model.

liquid drop model :-

⇒ This model proposed by Neil's Bohr, found symm similarities b/w nucleus & a liquid drop or as follows.

(i) The nucleus is supposed to be spherical in shape in a stable state just as a liquid drop. is spherical due to surface tension (force).

(ii) The force of surface tension acts on the surface of the liquid drop.

lly, potential barrier at the surface of the nucleus.

⇒ The density of a liquid drop is independent of its volume (or) surface area. Similarly, density of nucleus is independent of its volume (that means nucleus density is constant).

⇒ The inter molecular forces in a liquid (or) short range forces which interact only with nearest or immediate neighbours. Similarly, nuclear forces are short range forces protons & neutrons interact immediate neighbours. which leads that nuclear forces are short range forces & constant by finding energy for nucleon.

⇒ The molecules evaporate from a liquid drop & raising the temperature of the liquid due to increase of thermal agitation. lly, A nucleus collisions with nuclear projectiles gives a compound nucleus

which emits nuclear radiations. (Process of nuclear fission).

⇒ liquids exhibit phenomena of evaporation which compared with radio activity of nucleids.

⇒ Depends on Oscillation liquids drop breaks into components i.e. nucleides undergo the process of nuclear fission.

⇒ With the above similarities liquids & nucleids liquid drop model is proposed.

These concepts are defined mass of the nucleus & binding energy of nucleus.

⇒ This formula is called as Semi empirical mass formula.

Merits & Demerits shell model:

shell model describes the arrangement of nucleons in diff shells of nucleus.

→ The concept of magic numbers (2, 8, 20, 28, 50, 82, 126) can only be explained based on shell model.

→ shell model confirms a spin properties of nucleus.

→ Angular momentum, magnetic momentum in a nucleus is also a consequence of shell model.

→ electric quadrupole moment is also confirmed by shell model.

→ The concept of nuclear isomers is also confirmed by shell model.

→ Nuclear ^{stripping} b/w reactions explained by shell model.

→ Quadrupole moment of shell model is experimentally agreed.

3) The radius of holonium ^{radio} 165 is 7.731 Fermi. Calculate the radius of

80) $\text{Ho}^{165} \rightarrow R_1 \rightarrow A_1 = 165$
 $\text{He}^4 \rightarrow R_2 \rightarrow A_2 = 4$

$$R = R_0 A^{1/3}$$

$$\frac{R_1}{R_2} = \frac{A_1^{1/3}}{A_2^{1/3}}$$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$R_2 = R_1 \left(\frac{A_2}{A_1} \right)^{1/3}$$

$$7.731 \times \left(\frac{4}{165} \right)^{1/3}$$

$$\Rightarrow 2.238 \text{ fcmi} //$$

Radioactive Decay

Introduction: ^{radioactivity} was discovered by Henry Becquerel in 1896. He found that a uranium salt wrapped up in paper emitted certain penetrating radiations affected a photographic plate. This effect is spontaneous and not influenced by any external agency.

→ Further investigation Madame Curie, Pierre Curie, Rutherford showed that phenomenon was exhibited by heavy elements like uranium, polonium, radium, thorium etc.

→ Radioactivity involves the spontaneous transmutation of one element into other.

→ Radioactivity are of 2 types

1) Natural Radioactivity

2) Induced radioactivity / Artificial radio activity.

Natural Radioactivity: The phenomena of spontaneous emission of highly penetrating of heavy elements of atomic weight > 206 occurring in nature called Radioactivity

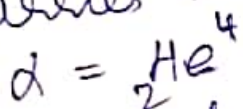
→ The elements which exhibit this property called Radioactive elements.

→ The atoms of radio active elements emit radiation is composed of three distinct kind of rays namely α -particle, β -particle and γ -radiation

Properties of three distinct kind of ray

properties of α -rays:

An α -particle is helium nucleus consisting of two protons & two neutrons. It carries 2 units of the charge.



α -particles shoot out from radioactive substance have high velocities ranging from 1.4×10^7 to 1.7×10^7 m/s. They move along st. line, their tracks can be observed in willson's cloud chamber

→ They produce intense ionisation in the gas through which they pass. Their ionising power is 100 times $>$ than β -rays & 10,000 times $>$ than that of γ -rays.

→ They exhibit effect of photographic plate when they strike on it.

→ When α -particles fall on fluorescent screens like barium thio-cyanide or zinc sulphide produce effect of fluorescence. α -rays deflected by electric & magnetic fields which shows α -rays contains charged particles

→ Nuclei of heavy elements like gold or uranium

→ α -particles produce heating effect are due to radioactive nature

→ e/m of α -particles found by Rutherford in terms of charge & mass compare to that of hydrogen concluded that mass of α -particles are four times that of hydrogen and charged the twice that of hydrogen. Thus, α -particle is the nucleus of Helium atom

Its e/m is 4.82×10^{-7} coulomb/kg

Mathematical formula:

$$BeV = \frac{mv^2}{\lambda} = \frac{e}{m} =$$

$$\Rightarrow \frac{e}{m} = \frac{dV^2}{\lambda}$$

Here λ = length of electric field applied

V = intensity of electric field
 d = deflection on photographic plate

v - speed of the α -particle

Geiger found charge of α -particle, mass of the α -particle in comparing with mass of hydrogen atom.

charge on the α particle (E) = 3.19×10^{-19} Coulomb
charge of electron (e) = 1.6×10^{-19}

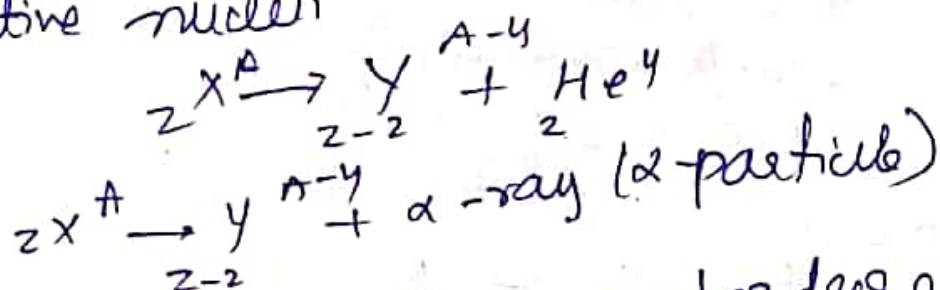
$$E = 2 \times e$$

$$\text{Mass of } \alpha \text{ particle } M = \frac{E}{m} = \frac{3.19 \times 10^{-19}}{4.82 \times 10^{-27}} \\ = 6.62 \times 10^{-27} \text{ kg} \\ \therefore \text{Mass of the } \alpha \text{ particle} = \frac{6.62 \times 10^{-27}}{1.67 \times 10^{-27}}$$

\Rightarrow 4) (In comparison with mass of hydrogen)

Basis of α -decay process.

An α decay is a process of emission of α -particles from disintegration of radioactive nuclei



In α decay, atomic number decreased by two units & mass decreased by ~~A-4~~ 4 units
Only heavy nuclei with $A > 200$ undergoes α -decay.

α -particles are helium nucleus before emission
 α -particles considered to be inside the nucleus.

→ Coulomb's law / Electrostatic applicable to α -particles when they are outside of the nucleus.

→ α -particles inside the nucleus, Coulomb's law does not hold good, due to ionisation

→ α -particles lose a large fraction of energy due to ionisation

→ Therefore, ionisation energy is the measure of related energy & range of the α -particle

Range of α -particles:

When α -particles pass through a gas it collides with atoms & molecules of the gas & ionises them it loses energy continuously by ionising atoms & molecules of the gas until its energy reduces to a value below the ionisation potential of the gas then the α -particle captures two electrons & becomes a neutral He atom.

The distance, the α -particle travels in the gas is called its range

Definition: The range of α particles customarily defined as the distance which these particles travel through

def of range of α -particles :-

The range of α -particles defined as the distance travels through a 76 cm of Hg (Barium) & 15°C temp. before they loose their energy to the extent that they no longer ionized that has no gas which is called the range of α -particles.

(or)

The distance through which α -particle travel through which before coming to rest

dependent factors :-

⇒ The range of α -particles depends on

(i) initial energy of α -particle.

(or)

initial velocity of α -particle.

(ii) Ionization potential of gas.

(iii) Nature of emitting radio active element.

(iv) Nature and pressure of gas. (or) nature of absorber.

⇒ In 1904, Bragg and Kleeman experimentally observed the factors of the range of α -particles and measured graphically 16 cm of Hg at 15°C of temp.

⇒ Geiger's Law :-

⇒ Geiger studied the relation b/w range (R) of an α -particle and its Velocity (V) of the emission found that range (R) is directly proportional to Cube of the Velocity of emission.

$$R \propto V^3$$

$$R = a \cdot V^3 \quad \text{--- ①}$$

here 'a' is called Geiger's Constant.

This relation holds good b/w the range of 3 cms to 8 cms.

⇒ As energy of the emitted particle is directly proportional to Square Velocity.

$$E \propto V^2$$

$$E = b V^2$$

$$\frac{E}{b} = v^2$$

$$\left(\frac{E}{b}\right)^{1/2} = v \quad \text{--- (2)}$$

Substituting eqn (2) in eqn (1)

$$R = a \left\{ \left[\frac{E}{b} \right]^{1/2} \right\}^3$$

$$R = a \cdot \frac{E^{3/2}}{b^{3/2}}$$

$$\Rightarrow R = \left(\frac{a}{b^{3/2}} \right) E^{3/2}$$

$$\boxed{R = k \cdot E^{3/2}} \quad \text{here}$$

$k = a/b^{3/2}$ is Const.

$$\boxed{R \propto E^{3/2}}$$

Above relation is called Geiger's law of energy.

Range of α -particle is directly prop to Cube root of energy.

23-10-19 S.N.Q.
* Geiger-Nuttal law :-

Different α -emitters emit α -particles with different energies. Hence, all the α -particles exist with different ranges.

Several experiments observed that α -emitters giving higher energy particles have shortest half life and lower energy particles have longer half lives.

Geiger & Nuttal measured the range of α -particles emitted by radio active elements and found the relationship b/w ranges and decay constants.

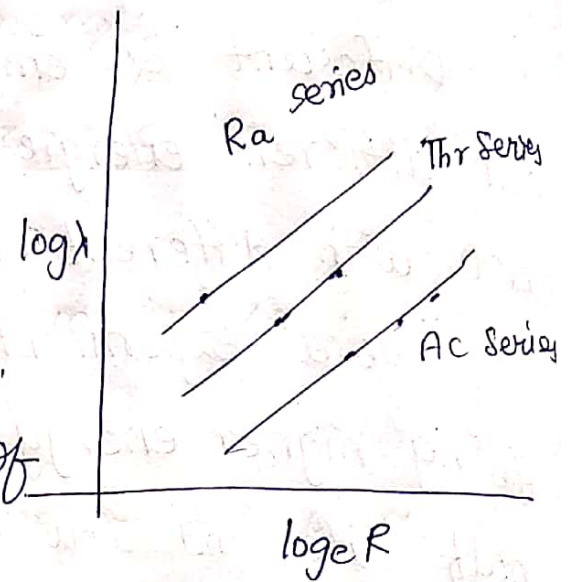
Mathematically, $\log \lambda = A + B \log R$.

where, ' λ ' disintegration const. A and B are constants for the given radio active

On x-axis $\log R$ for a different α -emitters in three radio active series drawn against $\log \lambda$ on y-axis gives 3 parallel st. lines. all the 3 parallel st. lines have diff. intercepts on $\log R$ which indicates Const. A with diff series while the value

of B is same for all series.

According to this disintegration const is high the range also high $\log \lambda$ since it depends on energy. Thus radio active substance of large decay const. emits.



high energy of α -particles.

Significance :- Geiger-nuttal law is helpful in determining decay const of very active for very shorts.

2) Geiger nuttal law is used to check the validity of any theory of decay.

limitations :- classical theory does not give satisfactory explanation of Geiger nuttal law.

Quantum physics successfully explained a proof of Geiger nuttal law by Gamow thus it is called Gamow's of α -decay.

24-10-19
* Gamow's theory of α -decay :-

* LAB.
What is Geiger Nuttal law? Derive

Geiger Nuttal law from Gamow's theory?
(or)

Explain in detail Gamow's theory of α -decay & obtain theoretical form of Geiger Nuttal law & how it is verified experimentally.

Ans :- Fundamental theory of α -decay :-

\Rightarrow A heavy nuclei with $A > 200$ undergoes α -decay. The α -particle emitted from a nuclei exist with discrete energy spectrum & consists of several groups.

\Rightarrow The most intensive of α -particles contain higher energy due to strong repulsive force expressed mathematically $F = \frac{1}{4\pi\epsilon_0} \frac{2(z-2) \cdot e^2}{r^2}$

here $(z-2)$ - atomic no. of daughter nucleus.
and r - distance between the charge $2e$ & e of α -particle.

The respective potential energy when α -particle is at the distance of r from the nucleus can be expressed as $V(r) = -\int_{\infty}^R \vec{F} \cdot d\vec{r}$

$$V(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{2(z-2)e^2}{R}$$

here R - nuclear radius at ' r ' > ' R '.

⇒ The max. potential energy

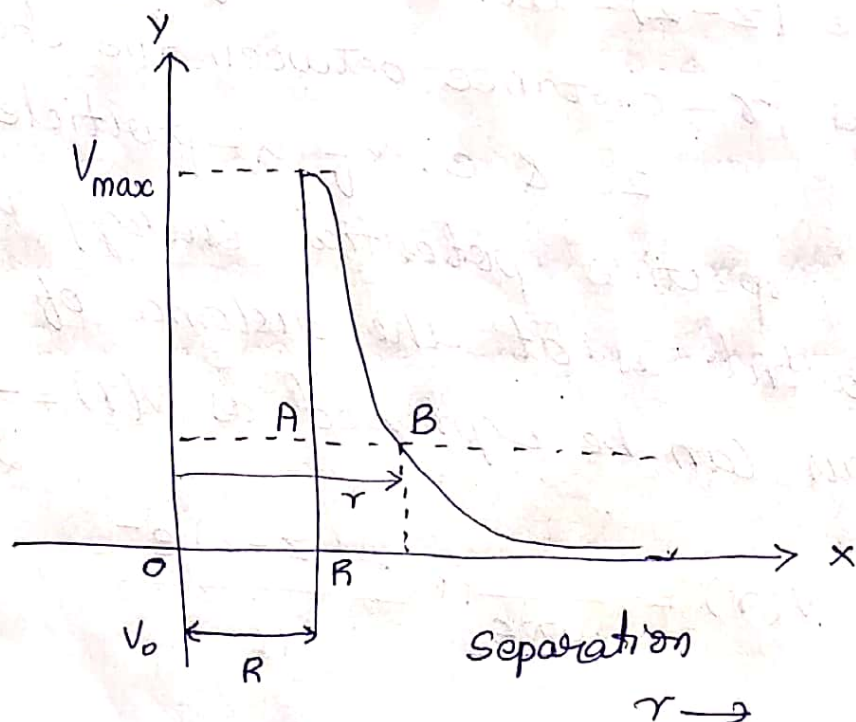
$$V_{\max} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2(z-2)e^2}{R_0 \cdot A^{1/3}}$$

Note : As P.E of the system increases with decrease of r .

Barrier height :- The ~~region~~ ^{region} of +ve potential called Barrier potential energy.

V_{\max} is considered as Barrier height.

⇒ within the nucleus short range of nuclear forces makes that α -particle is confined to nucleus which always attractive and its P.E is -ve. Thus, the nuclear forces are attractive within the nucleus this region of -ve P.E called Potential well.



⇒ In Case of Uranium potential barrier Uranium emits α -particle at 4 mega electron Volts.

⇒ classical electrx mechanics Cannot explain having an energy of 4 mega Volts having potential barrier of 26 meV.

⇒ \square

⇒ According to this it is possible for an α -particle to cross through the barrier named as tunneling effect.

* Gamow's theory of α -decay :-

The Basic notations of Gamow's theory

are

- (i) The α -particle exists as an entity within a heavy nucleus before emission.
- (ii) The α -particle is in const. motion & bounces back & forth b/w the barrier walls. during such collision with the walls of potential barrier. There is a definite

probability that α -particle leakage through the barrier.

\Rightarrow Once the α -particle leaks through the barrier it escapes from the nucleus with its kinetic energy and affected by Coulomb repulsive force.

derivation :-

Let V be the frequency with which the α -particle collides with the walls in order to escape from the nucleus.

Let P be probability of transmission in each collision.

Now the disintegration constant λ is given by

$$\lambda = VP \quad \text{--- (8.4)}$$

If there be only one α -particle collides with in the nucleus which moves back and forth along the nuclear diameter, then

$$V = \frac{v}{2R} \quad \text{--- (8.5)}$$

where V is the velocity of α -particle and R is the nuclear radius.

$$\log_e P = \frac{-2}{h/2\pi} \int_R^{R_1} \sqrt{2m \{V(r) - T\}} dx \quad \text{--- (8.6)}$$

where m = mass of α -particle.

$$\text{here } V(r) = \frac{2Ze^2}{4\pi\epsilon_0 r}$$

is the electrostatic potential energy of α -particle at a distance r from the nucleus of charge Ze . The charge on α -particle is $2e$.

R = Nuclear radius.

T = K.E of α -particle such that $T < V(r)$.

The region from R to r_1 is called the thickness of the barrier. we have,

$$\log_e P = \frac{-2}{h/2\pi} \int_R^{r_1} \sqrt{2m \left(\frac{2Ze^2}{4\pi\epsilon_0 r} - T \right)} dr$$

$$\text{At } r=r_1, T=V \quad \therefore T = \frac{2Ze^2}{4\pi\epsilon_0 r_1} \quad \text{--- (8.7)}$$

$$\therefore \log_e P = \frac{-2}{h/2\pi} (2mT)^{1/2} \int_R^{R_1} \left(\frac{r_1}{r} - 1 \right)^{1/2} dr \quad \text{--- (8.8)}$$

The integral.

$$\int_R^{\infty} \left(\frac{r_1}{r} - 1 \right)^{1/2} dr = r_1 \left[\cos^{-1} \left(\frac{R}{r_1} \right)^{1/2} - \left(\frac{R}{r_1} \right)^{1/2} \left(1 - \frac{R}{r_1} \right)^{1/2} \right]$$

The eqn. 8.8 Can be written as.

$$\log_e P = \frac{-2}{h/2\pi} (2mT)^{1/2} \otimes r_1 \left[\cos^{-1} \left(\frac{R}{r_1} \right)^{1/2} - \left(\frac{R}{r_1} \right)^{1/2} \left(1 - \frac{R}{r_1} \right)^{1/2} \right]$$

— (8.9)

The width of potential barrier is very large compared with nuclear radius

$r, r_1 \gg R$. Therefore,

$$\cos^{-1} \left(\frac{R}{r_1} \right)^{1/2} \approx \frac{\pi}{2} - \left(\frac{R}{r_1} \right)^{1/2}$$

$$\left(1 - \frac{R}{r_1} \right)^{1/2} \approx 1$$

$$\text{Hence } \log_e P = \frac{-2}{h/2\pi} (2mT)^{1/2} r_1 \left[\frac{\pi}{2} - \left(\frac{R}{r_1} \right)^{1/2} - \left(\frac{R}{r_1} \right)^{1/2} \right]$$

$$= -\frac{2}{h/2\pi} (2mT)^{1/2} r_1 \left[\frac{\pi}{2} - 2 \left(\frac{R}{r_1} \right)^{1/2} \right]$$

Substituting $r_1 = \frac{2ze^2}{4\pi\epsilon_0 T}$, we have

$$\log_e P = -\frac{2}{h/2\pi} (2mT)^{1/2} r_1 \frac{\pi}{2} + \frac{2}{h/2\pi} (2mT)^{1/2}$$

$$r_1 2 \left(\frac{R}{r_1} \right)^{1/2}$$

Substituting the values of Constants in eqn 8.10, we get

$$\log_e P = 2.97 Z^{1/2} R^{1/2} - 3.95 Z T^{-1/2} \quad (8.11)$$

where R is in Fermi and T is in MeV.

eqn. 8.4 can be written as

$$\log_e \lambda = \log_e V + \log_e P.$$

$$\log_e \lambda = \log_e \left(\frac{V}{2R} \right) + 2.97 Z^{1/2} R^{1/2} - 3.95 Z T^{-1/2}$$

$$2.303 \log_{10} \lambda = 2.303 \log_{10} \left(\frac{V}{2R} \right) + 2.97 Z^{1/2} R^{1/2} - 3.95 Z T^{1/2}$$

$$\therefore \log_{10} \lambda = \log_{10} \left(\frac{V}{2R} \right) + 1.29 Z^{1/2} R^{1/2} - 1.72 Z T^{-1/2} \quad (8.12)$$

In case of heavier nuclei the changes in atomic number and nuclear radius are negligible when compared to the changes in energy. The first term is almost same for heavier nuclei. So eqn. 8.12 is reduced to

$$\log_{10} \lambda = c + d T^{-1/2} \quad (8.13)$$

where c & d are constants.

$$c = \log_{10} \left(\frac{V}{2R} \right) + 1.29 Z^{1/2} R^{1/2} \quad \& \quad d = -1.72 Z$$

This eqn shows that the radioactive elements having lesser decay constant emit α -particles of greater energy (T) which is the Geiger and Nuttall law.