UNIT - 2

QUANTUM MECHANICS

matter waves :

- Debroglie's hypothesis.
- wavelength of matter waves } L.A.Q. (or) S.A.Q.
- Properties of matter waves.
- S.A.Q phase and group velocities.
- S.A.Q Davidson and Germer experiment.
- S.A.Q Standing Debroglie's waves in bohr's orbits.

2. Uncertainity principle:

- S.A.Q Uncertainity principle for position and momentum (x & p) / S.A.Q energy and time (E, t).
- L.A.Q Experiment illustrations Consequence of uncertainity principle.
- L.A.Q Determination of position of particle by gamma ray microscope.
- S.A.Q Position of electron in Bohr orbit
- S.A.Q Complimentary principle of Bohr's.

3. Schrodinger wave eqn. :

- L.A.Q Schrodinger time independent and time dependent wave eqns.
- physical significance S.A.Q wavefunction and its properties.

⇒ Application to harmonic oscillator.

25-7-19

* Introduction :-

A wave is characterised by a frequency (ν), wavelength (λ), velocity (v), amplitude (A) and intensity (I). which can be proved by several other experiments like interference, diffraction, electromagnetic radiation in visible, UV, X-rays proved the wave nature of matter.

⇒ A particle characterised by its position, velocity (v), momentum (P) and energy (E) for a particle (electron/photon), which can be proved by photo electric effect, x-rays absorption and Com relation verified that particle nature in the matter.

⇒ To understand matter behaviour as a partic (or) as a wave (duality) explained by Louis Debrogille in 1924. based on "nature loves symmetry" and matter also dual nature light.

Assumptions (or) Hypothesis of De-brogille :-

→ ~~A~~ French Physicist de-brogille in 1924 Considered to explain matter behaviour like a wave. According to him he introduced 'wave particle dualism'.

(1) Matter & Radiation are two fundamental forms in Nature. These two must be Symmetric.

→ As Radiation has both particle & wave nature matter also have Same dual nature (Particle & wave).

→ Similarity between Mechanics and optics related to fundamental principles retarding matter, momentum and energy which Supports dual nature.

→ When electron revolves in a stationary orbit based on Bohr's theory. Electron attributed wave nature in addition to particle nature i.e. electron as a matter exhibit dual nature

* Debrogille's wavelength (or) Wavelength of matter waves :- ~~According~~

According to De-Brogille's hypothesis, moving particle is associated with a wave which is known as Debroglee's wave.

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⇒ The wavelength of matter wave associated with a moving particle of momentum 'p' and its mass 'm' travelling with a velocity of a particle 'v' then expressed as $\lambda = \frac{h}{mv} = \frac{h}{p}$

Mathematical expression for Debrogille's wavelength:

I. A photon of light (Quantum) based on Plank's Quantum theory of electro magnetic radiation leads particle energy. $E = h\nu$. —— ①

where h = planck's Const. 6.634×10^{-34} J/sec.

ν = frequency. from the definition of Speed of light $c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda}$ —— ② Sub in eqn ② in ①

we get, energy of the photon, $E = \frac{hc}{\lambda}$. —— ③

From the Einstein's mass energy relation $E = mc^2$

from eqn ③ & ④.

$$mc^{\cancel{2}} = \frac{h\cancel{c}}{\lambda} \Rightarrow mc = \frac{h}{\lambda} \Rightarrow \boxed{\lambda = \frac{h}{mc}} \quad \text{—— ⑤}$$

here $p = mc$ (momentum of photon). eqn ⑤ is one form of Debrogillie's wavelength.

II. In Case of a material particle of mass 'm' moving with velocity 'v' then its momentum

$P = mv$.

⇒ The wavelength associated with the particle,

$\lambda = \frac{h}{mv} = \frac{h}{p}$ where $p = mv$.

= momentum of particle lyk $\bar{e}$.

Energy of matter partical :-

From classical physics K.E of the material particle $E = \frac{1}{2} mv^2$.

$E = p^2/2m$.

like $p = mv$

$p^2 = 2m\cancel{v}.E$

$p = \sqrt{2mE}$

$\lambda = \frac{h}{p}$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}} \quad —— (6).$$

⇒ When a charged particle 'q' is accelerated by potential difference 'V' then its kinetic energy $E = qV$. Therefore, De-broglie's wavelength $\lambda = \frac{h}{p}$

$\lambda = \frac{h}{\sqrt{2mE}}$

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

$V \rightarrow$ P.d eV or volt

$m \rightarrow$ mass of charged particles

$q \rightarrow$ charge of a particle.

λ = wave length or wave nature of charged particles

⇒ When a materials are at thermal equilibrium which depends temperature it can be expressed as $E = 3/2\, KT$.

∴ Debroglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

$$\lambda = \frac{h}{\sqrt{\cancel{2}m \cdot \frac{3}{\cancel{2}} \cdot K \cdot T}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$$

where T is absolute temperature.

S.A.Q

* Debroglie wavelength associated with electrons :-

Let us Consider a Case of electron (as a particle) of rest mass m_o. and charge (e) accelerated by a potential 'V' from the rest to velocity 'v'. The K.E of the electron = potential of electron ($\bar{e}$).

$$\Rightarrow \frac{1}{2} m_o v^2 = e.V \text{ (V- potential diff.)}$$

$$\Rightarrow m_o v^2 = 2.eV.$$

$$\Rightarrow v^2 = \frac{2.eV}{m_o} \Rightarrow v = \sqrt{\frac{2eV}{m_o}} \quad \text{—(1)}$$

from the debroglie's wavelength of matter

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equilibrium
expressed

waves $\lambda = \frac{h}{P} \Rightarrow \lambda = \frac{h}{m_o V}$ —— ②

Sub eqn ① in eqn②. we get

$$\lambda = \frac{h}{m_o\sqrt{\frac{2eV}{m_o}}}$$

$$\lambda = \frac{h(\sqrt{m_o})}{m_o\sqrt{e \cdot V_2}}$$

$$\lambda = \frac{h(\sqrt{m_o})}{(\sqrt{m_o})^2\sqrt{e \cdot V_2}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{m_o\, e\, V_2}}} \quad \text{—— ③}$$

rons :-

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in above eqn ③.

$h = 6.634 \times 10^{-34}$ J/sec.

$m_o = 9.11 \times 10^{-31}$ kg.

$e = 1.9 \times 10^{-19}$

$V = v$ (potential diff)

$$\lambda = \frac{6.634 \times 10^{-34}}{\sqrt{9.11 \times 10^{-31} \times 2 \times 1.9 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.26 \times 10^{-10}}{\sqrt{V}}$$

$$\lambda = \frac{12.26}{\sqrt{V}}\ \mathring{A} \quad \text{—— ④}$$

er

The above eqn ④ is the wavelength associated with electron. accelerated which gives particle electron behaves as a wave.

* <u>Properties of Matter waves</u> :-

1) Lighter the particle, greater is the wavelength associated with it.

$$\lambda \propto \frac{1}{m}.$$

2) Smaller the velocity of the particle greater the wavelength associated with it.

$$\lambda = \frac{h}{mv}.$$

⇒ when $v = 0$, $\lambda = \infty$ i.e. wave become indeterminate.

If $v = \infty$, $\lambda = 0$ i.e. matter waves are generated by motion of particles.

The waves produced whether the particles are charged (or) uncharged which gives waves are non-electromagnetic but electromagnetic waves are produced at $V = C \neq 0$.

- The velocity of matter wave depends on velocity of matter particle.
- The Velocity of matter wave is greater than Velocity of light.
- Explanation :- $E = h\nu$

$$E = mc^2$$

$$h\nu = mc^2$$

$$\nu = \frac{mc^2}{h}.$$

The wave velocity (ω) or ($W = \nu\lambda$) — (2)

sub eqn (2) in (1) $\Rightarrow W = \left(\frac{mc^2}{h} \cdot \lambda\right)$.

$$W = \frac{mc^2}{h} \times \frac{h}{mV}$$

$$W = \frac{c^2}{V.}$$

c^2 — light velocity

V — particle velocity

wave Velocity — W

If $V = c \Rightarrow W = \frac{c^2}{c} = c$

$\therefore$ ~~wavelen~~ velocity = light velocity.

$$W = c.$$

Conclusion :- As particle velocity cannot exceed c. Hence, wave velocity is greater than Velocity of light. This can be known as Phase Velocity and group velocity.

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⇒ In a Same experiment wave & particle asspects of moving bodies can never get together.

⇒ Wave nature of matter introduces idea about uncertainity.

30-7-19 L.A.Q

* Phase (or) Group Velocities :- (wave velocity)

Q. Explain the terms of wave velocity (or) phase velocity and group velocity.
Obtain an expressions and relation b/w them.

Def :- Wave Velocity (or) Phase Velocity :-
when a monochromatic wave of frequency μ (or) n and wavelength λ travells through a medium then its velocity in the medium Called as wave velocity.

Mathematical expression :-
Consider a wave of displacement $y = a \sin(wt - Kx)$. here a = amplitude, w = angul frequency, (or) angular velocity, K = propogation Constant.

The ratio of angular frequency to the propogation constant Called as wave Velocity. denoted by V_p. $V_p = \frac{w}{K}$ —— ①

For the wave $(\omega t - kx)$ is the phase of wave motion whose value is Const. (for the property of wave phase difference is Const).

$$\therefore \omega t = kx \quad (\text{or}) \quad (\omega t - kx) = \text{Const.} \quad \text{--- (2)}$$

diff. on b.s w.r.t time.

$$\frac{d}{dt}(\omega t - kx) = \frac{d}{dt}(\text{Const.})$$

$$\omega \cdot \frac{dt}{dt} - k \cdot \frac{dx}{dt} = 0.$$

$$\omega(1) - k(V_p) = 0.$$

$$\omega - \cancel{k}\left(\frac{\omega}{\cancel{k}}\right) = 0 \quad (\because \text{eqn (1)})$$

$$\Rightarrow \omega(1) - k \cdot \frac{dx}{dt} = 0.$$

$$\omega = k \cdot \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{\omega}{k}.}$$

$$\boxed{\frac{dx}{dt} = V_p = \text{phase Velocity} = \frac{\omega}{k}.}$$

with which the

Hence, it is Called phase Velocity.

Group Velocity :-

It is denoted by V_g. Group Velocity is the velocity of different pulse (slight change in frequency one another) with which the energy in the group is transmitted.
Mathematically denoted as $\frac{dw}{dt}$.

Mathematical expression :-

Consider two waves of Same amplitude (a) and slite different angular velou frequencies of w and w' and phase velocity v and v'.
From the classical physics the displacement of two waves expressed as

$$y_1 = a \sin(wt - Kx) \quad \text{--- (1)}$$

$$y_2 = a \sin(w't - K'x) \quad \text{--- (2)}$$

where K and K' are propogation Constants.

Resultant of two waves expressed as,

$$y' = y_1 + y_2.$$

$$y = a \sin(wt - Kx) + a \sin(w't - K'x).$$

$$\boxed{\sin A + \sin B = -\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)}$$

$$y = 2a \cos\left[\left(\frac{w-w'}{2}\right)t - \left(\frac{K-K'}{2}\right)x\right] \sin\left[\left(\frac{w-w'}{2}\right)t - \left(\frac{K+K'}{2}\right.\right.$$

$$y = 2a\cos\left[\left(\frac{d\omega}{2}\right)t - \left(\frac{dK}{2}\right)x\right]\sin\left[\omega t - Kx\right] \quad \text{--- (3)}$$

$$\frac{\omega+\omega'}{2} \approx \omega \quad , \quad \frac{K+K'}{2} \approx K.$$

The above eqn (3) gives a wave of angular frequency (ω).

The phase velocity of resultant wave are expressed as $V_p = \frac{\omega}{K} \approx$ Same as each composing wave. Thus, Amplitude of resultant wave is modified. This can be expressed

$$A = 2a\cos\left[\left(\frac{d\omega}{2}\right)t - \left(\frac{dK}{2}\right)x\right]$$

$$A = 2a\cos\frac{d\omega}{2}\left[t - \frac{dK}{\cancel{2}\cdot\frac{d\omega}{\cancel{2}}}\cdot x\right]$$

$$A = 2a\cos\left(\frac{d\omega}{2}\right)\left[t - \frac{dK}{d\omega}\cdot x\right].$$

$$A = 2a\cos\left(\frac{d\omega}{2}\right)\left[t - \frac{x}{\left(\frac{d\omega}{dK}\right)}\right].$$

$$A = 2a\cos\left(\frac{d\omega}{2}\right)\left[t - \frac{x}{V_g}\right].$$

here $V_g = \frac{d\omega}{dK}$.

$$\boxed{V_g = \frac{\omega-\omega'}{K-K'}} \quad \text{--- (4)}$$

Relation b/w Group Velocity and Wave Velocity:

As we know, $V_p = \frac{\omega}{k} \Rightarrow \omega = V_p \cdot k$.

diff. on b.s

$$d\omega = d(V_p \cdot k)$$

$$d\omega = V_p \cdot dk + k \cdot dV_p$$

$$\frac{d\omega}{dk} = V_p + \frac{k \cdot}{dk} dV_p$$

$$V_g = V_p + k \cdot \frac{dV_p}{dk} \quad \text{--- } (5)$$

$$V_g = V_p + \left(\frac{2\pi}{\lambda}\right) \cdot \frac{dV_p}{\left(\frac{2\pi}{d\lambda}\right)}$$

$$V_g = V_p + \left(\frac{1}{\lambda}\right)\left(\frac{dV_p}{\frac{1}{d\lambda}}\right)$$

$$V_g = V_p + \left(\frac{d\lambda}{\lambda}\right) dV_p.$$

$$\Rightarrow \boxed{V_g - \left(\frac{d\lambda}{\lambda}\right) dV_p = V_p.} \quad \text{--- } (6).$$

from eqn (5)

$$V_g = V_p + k \cdot \frac{dV_p}{dk}.$$

$\left[\because k = \frac{2\pi}{\lambda}\right.$

$\left.\lambda = \frac{2\pi}{k}\right]$.

$$d\lambda = 2\pi\left[d\left(\frac{1}{k}\right)\right]$$

$$d\lambda = \frac{-2\pi}{k^2} \cdot dk$$

$$dk = \frac{-k^2}{2\pi} \cdot d\lambda$$

$$\cancel{V_g = V_p + k \cdot \frac{dV_p}{dk} \cdot \frac{d\lambda}{d\lambda}}$$

$$V_g = V_p + \frac{k \cdot dV_p}{\left(\frac{-k^2}{2\pi}\right) d\lambda}.$$

$$V_g = V_p - \frac{dV_p}{k} \times \frac{2\pi}{d\lambda}$$

$$V_g = V_p - \left(\frac{2\pi}{k}\right) \cdot \frac{dV_p}{d\lambda}.$$

$$\Rightarrow \left(\because \frac{2\pi}{k} = \lambda\right)$$

$$\boxed{V_g = V_p - \lambda\left(\frac{dV_p}{d\lambda}\right)} \quad \text{—— (7)}$$

The above eqn (7) is more appropriate then eqn(6). ~~thus~~. ~~eqn(6)~~ has no significance

$\therefore$ eqn (7) $V_g = V_p - \lambda\left(\frac{dV_p}{d\lambda}\right)$ Called as relation b/w group velocity and phase velocity.

~~* Relation b/w~~

13-8-19

* Standing Debroglie's waves in borh's orbits :-

This is also Called as Wave Mechanical atom model.

16-8-19 According to Debroglie's interpretation the electron of mass 'm' moving with velocity 'V' associated with wavelength 'λ' expressed as

$$\lambda = h/mV \quad \text{—— (1)}$$

where h = planck's Const.