The eqn ① gives wave character of electron which is limited in a specified permissible orbit in an atom.

→ Explanation :-

According to Bohr electron in an atom revolves in a circular stationary orbit around the nucleus. In this orbit electron neither loose nor radiate its energy. Further, the electron wave adjusted around 'n' orbit those circumference $(2\pi r)$ is an integral multiple of wavelengths which means stationary orbit of an electron contain a complete wave of electron.

Mathematically $2\pi r = n\lambda$ —— ②

here $n = 1, 2, 3 \ldots$

Sub eqn ① in eqn ②, we get

$$2\pi r = n\left(\frac{h}{mv}\right)$$

$$\Rightarrow mvr = n \cdot \frac{h}{2\pi}$$

$$\therefore \boxed{L = n\hbar} \quad (\hbar = h/2\pi) \quad \text{—— ③}$$

The eqn ③ is called Bohr quantisation slide.

n. Debrogilie wavelength and Circumference of wave orbit :

According to wave theory, the radius of n^{th} orbit is expressed as $r_n = \frac{n^2 h^2 \epsilon_0}{\pi . m e^2}$

the Circumference of the orbit

$$2\pi r_n = 2\cancel{\pi} \frac{n^2 h^2 \epsilon_0}{\cancel{\pi} m . e^2}$$

$$2\pi r_n = \frac{2 n^2 h^2 \epsilon_0}{m . e^2} \quad \text{--- (4)}$$

The velocity of electron in n^{th} orbit,

$$V_n = \frac{e^2}{2 n h \epsilon_0} \quad \text{--- (5)}$$

Then linear momentum of electron

$$m V_n = \frac{m . e^2}{2 . n . h . \epsilon_0}$$

The debroglie's wavelength of electron in n^{th} orbit is expressed as $\lambda_n = \frac{h}{m . V_n}$

$$\lambda_n = \frac{2 . h (n . h \epsilon_0)}{m . e^2}$$

$$\lambda_n = \frac{2 n h^2 \epsilon_0}{m . e^2} \quad \text{--- (6)}$$

Comparing eqn (4) and (6), we get

$$2\pi r_n = n . \lambda_n \quad \text{--- (7)}$$

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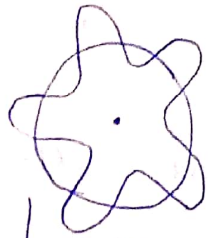
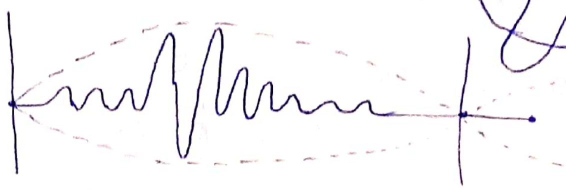
$$\Rightarrow \quad 2\pi r_1 = \lambda_1$$
$$2\pi r_2 = \lambda_2 \; 2\lambda_2$$
$$2\pi r_3 = 3\lambda_3$$
$$\vdots$$

The Circumference of first bohr orbit is equal to its wavelength. thus, the Circumference of the n^{th} orbit is equal to n times of its wavelength. Thus Bohr atom model of first postulate dual nature of matter. Hence electron in the orbit behaves like a wave.

* ~~Un~~ wavepacket :- wave particle association behaviour.





* Uncertainity principle :-

⇒ Debroglie's Concept leads to uncertainity principle. The material particle exhibits particle nature and wave nature but two natures Cannot be explained.

⇒ Debroglie Concept of matter waves is fundamental for the development of quantum mechanics or micro system mechanics.

, wave property of electron is utilized to design electron microscope 1000 times better resolution except other.

, The neutron diffraction of technique frequently used the structure of matter.

, Dual character posses by matter leads to develop the uncertainity principle.

* Heisenberg uncertainity principle :-

→ In 1987, German physist Werner heisenberg proposed a principle known as a principle of Indeterminancy and Uncertainity principle.

This principle sets fundamental limits to the simultaneous determination of pair of variables such as position & momentum, energy & time, Angular momentum and angular displacement.

* Position momentum uncertainity :-

1. It is impossible to measure both the position and momentum of a particle simultaneously with desired degree of accuracy. (unlimited accuracy).

(or)

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2. It is impossible to Specify precisely and Simultaneously the value of both members of particular pairs of physical variables that describes the behaviour of an atomic System.

(or)

3. The order of magnitude of uncertainities in the knowledge of two variables must be atleast planck's Const. H.

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Mathematical expression :-

According to the principle the position & momentum of a particle (electron) Cannot be determine Simultaneously to any desired degree of accuracy.

⇒ let 'Δx' be the error indetermining position and 'ΔP' error indetermining the momentum at the Same instant, then the principle Can be expressed mathematically as a product of Δx.Δp approximately equal to $\frac{h}{2\pi}$ (or)

$$\Delta x \cdot \Delta P \geq \frac{h}{2\pi}$$

$$\boxed{\Delta x \cdot \Delta P \geq \hbar} \quad \left(\because \hbar = \frac{h}{2\pi}\right) \quad \text{——} (1)$$

since the product of errors in position and momentum (Δx and ΔP) is the order of planck's Constant. which means Δx is large ΔP is Small.

$\Delta x \rightarrow$ Small , $\Delta P \rightarrow$ large.

which means that if one quantity measure accurately other quantity becomes less accurate.

⇒ Therefore, any instrument Cannot be measured the quantities more accurately than predicted by heisenberg uncertainity principle or indeterminancy.

* Energy – time Uncertainity :-

⇒ The uncertainity relation Can also be obtained for other pairs of variables like Kinetic energy & time from clasical physics,

$$K.E \Rightarrow E = \frac{P^2}{2m} \quad \text{——} ①.$$

$\longrightarrow$ diff. on B.S. we get

$$\Delta E = \frac{\Delta P^2}{2m}.$$

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{P^2}{2m}\right]$$

$$\frac{dE}{dt} = \frac{1}{2m}\left[\frac{d}{dt}(P^2)\right].$$

$$\frac{dE}{dt} = \frac{1}{\cancel{2}m}\left[\cancel{2}P.\frac{dP}{dt}\right]$$

$$dE = \frac{P}{m} \cdot dP \qquad \Rightarrow \Delta P = \frac{\Delta E\,(\cancel{2}m_0)}{\cancel{2}P}$$

$$dE = \frac{mv}{m} \cdot dP \qquad \Delta P = \frac{\Delta E\, m_0}{P} \quad \text{—— (2)}$$

$$dE = v \cdot \frac{m\,\Delta x}{\Delta t}$$

$$dE = mv \cdot \frac{\Delta x}{\Delta t}$$

$$\Delta t \cdot dE = m\frac{x}{t}\,\Delta x$$

$$\left(\because \approx \frac{h}{2\pi} \approx \hbar\right)$$

$$\Delta E \cdot \Delta t \approx \hbar$$

$$\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$$

$$\uparrow \Delta E \approx \frac{h}{2\pi} \cdot \Delta t \uparrow$$

$\Rightarrow$ From heiesenberg uncertainity principle

$$\Delta v = \frac{\Delta x}{\Delta t}$$

$$\Delta x = \Delta v \cdot \Delta t \quad \text{—— (3)}$$

from eqn (2) & (3)

$$\Delta P \cdot \Delta x = \frac{\Delta E\, m_0}{P} \cdot \Delta v \cdot \Delta t$$

$$\Delta P \cdot \Delta x = (\Delta E \cdot \Delta t) \cdot \frac{\Delta v \cdot m_0}{P}$$

$$[\Delta P \cdot \Delta x]\,\frac{P}{\Delta v \cdot m_0} = \Delta E \cdot \Delta t$$

$$[\cancel{\Delta v} \cdot \cancel{m_0} \cdot \Delta x]\,\frac{P}{\cancel{\Delta v} \cdot \cancel{m_0}} = \Delta E \cdot \Delta t \Rightarrow \Delta x \cdot P = \Delta E \cdot \Delta t$$

$$\frac{h}{2\pi} \approx \Delta E \, \Delta t$$

$$\boxed{\Delta E \cdot \Delta t \approx \hbar}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

⇒ Thus the product of uncertainities in energy and time is always greater than or equals to $\hbar$ or $\frac{h}{2\pi}$ (or) $\frac{h}{4\pi}$. Simillarly, if Δj and $\Delta \theta$ are the uncertainities in angular momentum and angular displacement.

⇒ Thus uncertainity principle is universal and holds for all pairs of Conjugate Variables, the product of whose dimension yeilds the dimensions of action.

⇒ Heisenberg principle implies that in values of principle measurements, probability of exactness.

⇒ According to classical phy it is impossible to find a very Small but finite probability of accurance.

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* Experimental illustrations in Support of Uncertainity principle :-

(1) Determination of position with γ-ray microscope ;

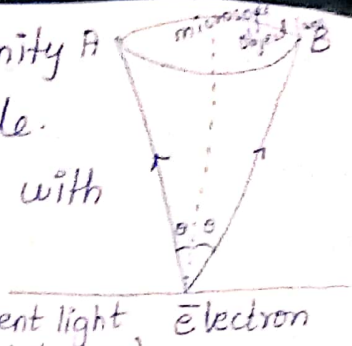
(2) Diffraction of a beam of electrons by a single slit.

(1) γ-ray :- Consider a case of measurement of the position of a particle (electron) in the field of γ-ray microscope. A very high resolving power of microscope electron can be observed if atleast one photon is Scattered by it into the microscope lens. which can be expressed by the relation $\Delta x = \frac{\lambda}{2\sin\theta}$ —— ①.

here λ = wavelength of light

θ = Semi Vertical angle of the Cone of light.

Δx distance b/w 2 points uncertainity in determining position of the particle.



> The incoming photon will interact with electron through a Compton effect which can be found in microscope (photon) with in the angle 2θ.

> The momentum imp by to the electron during the collision is in the order of $P \approx \frac{h}{\lambda}$.

> The Component of this momentum along OA is $\frac{h \sin\theta}{\lambda \cancel{\sin\theta}}$ and along OB is $\frac{h}{\lambda} \sin\theta$.

> $\therefore$ Uncertainity in the momentum in the direction of x we write ΔP or Δx.

$$\Delta P_x = \frac{h}{\lambda} \sin\theta - \left(-\frac{h}{\lambda} \sin\theta\right)$$

$$\Delta P_x = \frac{2h}{\lambda} \sin\theta.$$

> From heisenberg's uncertainity $\Delta x \cdot \Delta P_x \approx \frac{h}{2\pi}$.

$$\Delta x \cdot \Delta P_x = \frac{\cancel{\lambda}}{\cancel{2} \cancel{\sin\theta}} \times \frac{\cancel{2}h}{\cancel{\lambda}} \cancel{\sin\theta}$$

$$= h \qquad \Delta x \cdot \Delta P_x = \frac{h}{2\lambda}.$$

> A more satisficated approach shows

Conclusion :-

> Thus the product of uncertainity in momentum and position approaches to planck's Constant.

> which proves heisenberg uncertainity position & momentum.

(2) A Beam of electron transmitted through a Slit and recieved in a photography plate 'p' Kept at a distance from the slit then electron process through the slit there by produce diffraction pattern on the Screen.

> let 'Δy' be the position of any electron on the plate which is equal to width of the slit.

> let 'λ' be the wavelength of the electron and 'θ' be the angle of deviation. Corresponding to first minimum.

> From Single Slit diffraction minima gives

$$d \sin\theta = n\lambda$$

$$\Delta y \cdot \sin\theta = 1 \cdot \lambda \quad (\because n = 1,\ d = \Delta y).$$

$$\Delta y = \frac{\lambda}{\sin\theta} \qquad \text{——— (1)}$$

> eqn(1) gives uncertainity in determining position of electron along Y-direction.

> Initially the electrons are moving along the X-axis no Component of momentum along Y-axis.

> After diffraction at the slit electron deviated exist Component of momentum along 'y'

which is $P \cdot \sin\theta$.

As the electron exist along y - Component with in the reach of θ and $-\theta$ then the Component of the momentum of the electron exist any where b/w $P \cdot \sin\theta$ and $-P \sin\theta$.

$\therefore$ Uncertainity in 'y' Component momentum of electron $\Delta P_y = P \sin\theta - (-P \sin\theta)$.

$$\Delta P_y = -2p \sin\theta \qquad \left[\begin{array}{l} \lambda = \frac{h}{P} \\ P = \frac{h}{\lambda} \end{array} \right]$$

$$\Delta P_y = \frac{2h}{\lambda} \sin\theta \quad \text{———} \quad (2)$$

⇒ Multiplying the eqn (1) and (2), we get,

$$\Delta y \cdot \Delta P_y \approx \frac{\lambda}{\sin\theta} \times \frac{2h}{\cancel{\lambda}} \cancel{\sin\theta}$$

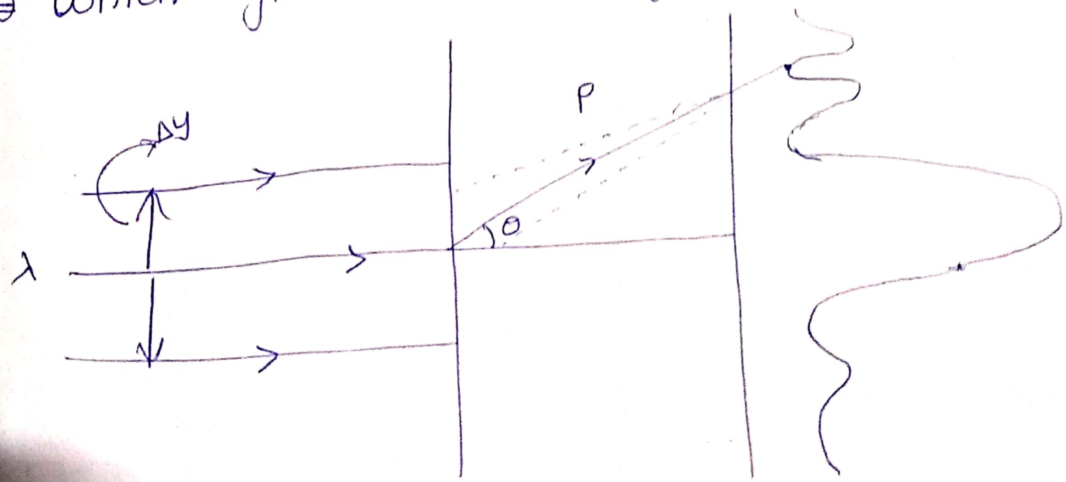
$$\Delta y \cdot \Delta P_y \approx 2h.$$

⇒ The more satisfying way,

$$\Delta y \cdot \Delta P_y \geq \frac{h}{4\pi}$$

$$\Delta y \cdot \Delta P_y \geq \hbar.$$

⇒ which gives heisenberg uncertainity principle.



* Consequence of uncertainity relation position of electron in Bohr orbit :

Calculating the radius of Bohr's first orbit :

If Δx and ΔP are uncertainities in position of the electron in the first orbit then according to uncertainity $\Delta x \cdot \Delta P \geq \frac{h}{4\pi}$.

$$\Rightarrow \Delta P \geq \frac{h}{4\pi(\Delta x)} \quad \text{——— (1)}$$

The uncertainity in kinetic energy ΔE of the electron $\Delta E = \frac{1}{2} m (\Delta V)^2$

$$\geq \frac{1}{2} \frac{m^2 (\Delta V)^2}{m}$$

$$\geq \frac{1}{2} \left(\frac{(\Delta P)^2}{m} \right)$$

$$\Delta E \geq \frac{(\Delta P)^2}{2m} \quad \text{——— (2)}$$

Substituting (1) × (2)

$$\Delta E \geq \left(\frac{h}{4\pi} \right)^2 \frac{1}{(\Delta x)^2 \cdot 2m}$$

$$\Delta E \geq \frac{h^2}{16\pi^2 \cdot 2 \cdot m \, (\Delta x)^2} \quad \text{——— (3)}$$

The uncertainity in the potential energy of the electron expressed as

(Potential energy) $\Delta V \geq \frac{-Ze^2}{\Delta x}$ —— (4)

The uncertainity in the total energy

$$\Delta E \geq \Delta E_K + \Delta V$$

$$\Delta E = \frac{h^2}{16\pi^2 . 2m (\Delta x)^2} + \left(\frac{-Ze^2}{\Delta x}\right) \quad \text{—— (5)}$$

The uncertainity in the energy will be minimum (maximising accuracy of finding in the electron in Bohr orbit).

$$\frac{d(\Delta E)}{d(\Delta x)} = 0.$$

$$\frac{d^2(\Delta E)}{d^2(\Delta x)} = +ve.$$

$$\frac{d(\Delta E)}{d(\Delta x)} \geq \frac{-2h^2}{16\pi^2 . 2m(\Delta x)^3} + \frac{Ze^2}{(\Delta x)^2}.$$

$$\Rightarrow \frac{+2h^2}{16\pi^2 2m(\Delta x)^2} \approx \frac{Ze^2}{(\Delta x)^2}$$

$$= \frac{h^2}{16\pi^2 m(\Delta x)}.$$

$$\Delta x = \frac{h^2}{16\pi^2 m z e^2} \quad \text{---} \quad (6).$$

As we Calculate or differentiating eqn (6) gives out positive and hence eqn (6) represents minimum. this minimum energy, the electron must be atleast Δx away from the nucleus (radius of the bohr).

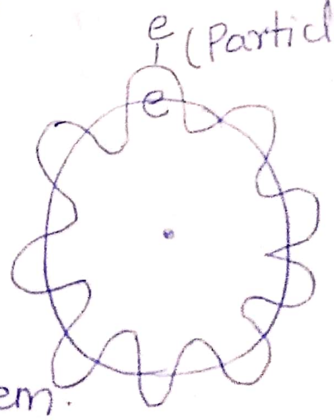
∴ The energy of the electron in the first orbit of the bohr is minimum.

∴ The radius of the first bohr of orbit is $\gamma = \Delta x \approx \frac{h^2}{2mze^2}$ $\left(\hbar = \frac{h}{2\pi}\right)$.

* Complimentary principle of Bohr :-

Statement :-

According to Complimentary principle the wave and particle nature of matter and light are Complimentary rather than Contradictory.



⇒ that means both aspects of necessary to have a Complete picture of the System.

⇒ This is also called as 6^{th} bohr orbit.